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AND

DISCUSSIONS

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

AN INVESTIGATION OF STEEL RIGID FRAMES

By Inge Lyse,[1] M. Am. Soc. C. E., and W. E. Black,[2] Jun. Am. Soc. C. E.

Synopsis

Tests on two riveted-steel, rigid-frame models (scale, 4 to 1) are described in this paper. In one frame, the knee sections were approximately square, having a sharp reentrant angle at the inner corner. The other frame had a large circular fillet at the inside corner of the knee. The frames were tested chiefly as two-hinged structures under working loads.

In general, the structural behavior of the two rigid frames was in accordance with conventional theory. At the knees of both frames, however, the normal stress distribution departed markedly from the usual straight-line relationship. In the square knee, a concentration of stress existed at the inner corner but was found to be of minor importance. In the curved knee, compressive stresses in the flange of the curved fillet were considerably greater than those computed by either the straight-beam or the curved-beam theories. Furthermore, a transverse variation of the stress in the outstanding legs of the curved flange angles increased the high compressive stresses in the curved knee. On the basis of the test results, recommendations for the analysis and design of each type of rigid frame have been made and are presented herewith.

Introduction

During the decade 1925–1935 there was a growing appreciation of the many structural and esthetic advantages of the rigid-frame type of construction, particularly as applied to short-span bridges. However, due to the lack of available information regarding the stress distribution in this type of structure, especially at the knee section, the steel rigid frame has been viewed with some concern by many engineers.

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **March 15, 1941.**

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In order to remedy this situation, the American Institute of Steel Construction (A. I. S. C.) initiated two experimental investigations of the structural behavior of steel rigid frames. One was conducted on rigid-frame knee sections by the National Bureau of Standards at Washington, D. C.;[3,4,5] and the second investigation, which is presented for discussion in this paper, was conducted on complete one-quarter size models, and was made possible by the establishment of a cooperative research fellowship by the A. I. S. C. and Lehigh University at the Fritz Engineering Laboratory, Bethlehem, Pa.

In the investigation conducted at the National Bureau of Standards, the primary purpose was to determine the stress distribution at the knee section. At Lehigh University, two complete rigid-frame models were tested to secure a check upon the stress distribution obtained by the testing of knee specimens only, and also to permit the observation of other important data regarding the frame as a whole. The focal point of interest was the knee section, although such subjects as movement of foundations and accuracy of conventional methods of analysis and design were also studied.

Test Program

The investigation was planned with the view of studying the behavior of the frames when hinged at the supports. The test program was laid out with the following objectives in mind:

A. Determination of stress distribution in the knee section (1—principal stresses and maximum shears, 2—normal stresses and shears on arbitrary sections);

B. Effect of the stress distribution in the knee upon the behavior of the frames as a whole (1—on horizontal reaction, 2—on stresses away from the knee, and 3—on deflections);

C. Effect of simulated foundation slippage (1—on horizontal reaction, 2—on stresses away from the knee, and 3—on deflections);

D. Determination of restraint provided by flat-plate base;

E. Comparison between experimental data and calculated results.

Test Specimens

In order to compare the results of this investigation with similar tests at the National Bureau of Standards,[3,4,5] the shapes of the knee sections of the two model frames were made respectively similar to two of the specimens tested at the Bureau.

The chief point of difference between the two models was the shape of the knee. One, referred to as the square-knee frame, had a sharp reentrant angle of slightly more than 90° at the inside corner of the knee; whereas the other, designated as the curved-knee frame, had a large circular fillet at the inside of the knee. The details of the two frames are shown in Fig. 1. The method

[3] "Strength of a Riveted Steel Rigid Frame Having Straight Flanges," by Ambrose H. Stang, Martin Greenspan, and William R. Osgood, M. Am. Soc. C. E., *Research Paper No. 1130, Journal of Research*, National Bureau of Standards, Vol. 21, 1938, p. 269.

[4] "Strength of a Riveted Steel Rigid Frame Having a Curved Inner Flange," by Ambrose H. Stang, Martin Greenspan, and William R. Osgood, *Research Paper No. 1161, loc. cit.*, p. 853.

[5] "Strength of a Welded Steel Rigid Frame," by Ambrose H. Stang and Martin Greenspan, *Research Paper No. 1224, loc. cit.*, Vol. 23, 1939, p. 145.

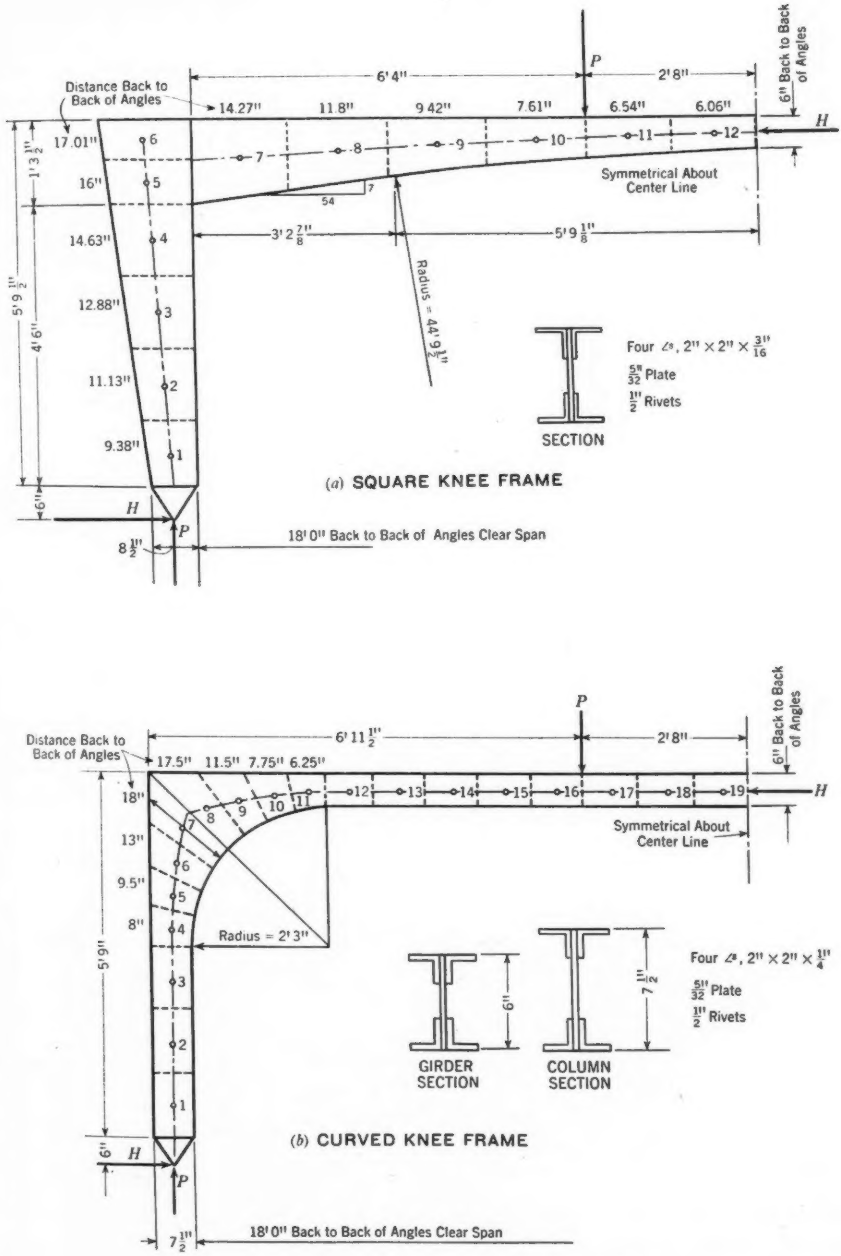


FIG. 1.—DETAILS OF MODEL FRAMES (SCALE RATIO, 4 : 1)

of fabricating the square knee, showing that the web does not extend through the vertical section at the inside corner of the knee, is illustrated in Fig. 2. The size of the models was chosen to fit the testing machine available, and the applied loads were designed to produce permissible working stresses in the models.

The square-knee frame (Fig. 1(*a*)) was considered to be reduced from an imaginary prototype with a clear span of 72 ft and was designed as the middle one of three such frames spaced 15 ft apart, with framed floor beams and stringers supporting a 36-ft roadway with an H-20 loading. The linear dimensions of the model were one fourth and the cross-sectional areas approximately one sixteenth of those of the prototype. The curved-knee frame (Fig. 1(*b*)) was later designed for the same conditions.

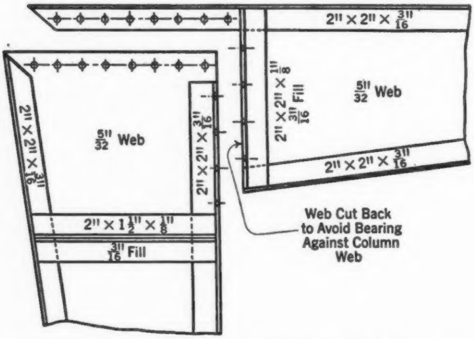


FIG. 2.—FABRICATION DETAILS OF A SQUARE-KNEE FRAME

Due to the small size of the frames, it was considered advisable to have them fabricated at a plant equipped for small-scale iron work rather than at an ordinary structural shop. The square-knee frame was fabricated by the Bethlehem Fence Works, in Bethlehem, and the curved-knee frame by the Allentown Iron Works, in Allentown, Pa.

In general, the workmanship of the two frames was satisfactory. Over-all dimensions and depths of the sections were sufficiently accurate to permit use of the nominal values in all but a few instances. One objectionable feature was

TABLE 1.—TENSILE TESTS OF STEEL COUPONS

Coupons cut from	(a) Square-Knee Frame: Stresses in Kips[a] per Square Inch — Modulus of elasticity	Yield point	Tensile strength	Elongation in 8 in.	(b) Curved-Knee Frame: Stresses in Kips[a] per Square Inch — Modulus of elasticity	Yield point	Tensile strength	Elongation in 8 in.
Angles	28,400	43.1	62.7	24.1	29,370	39.5	59.3	30.0
Plates	28,570	45.8	54.4	22.0	29,400	....[b]	63.6	15.7

[a] Kips = "kilo-pounds" = thousand pounds. [b] No definite yield point.

found, however, in the square-knee frame. At the intersection of the compression flanges at both knees, where tight bearing should be obtained, small gaps existed. It was considered advisable to fill these gaps with shims that were tackwelded in place, but which produced tight bearing only along the

outstanding legs of the girder flange angles. Thus, a loose fit was eliminated at the expense of a concentration of stress at the bearing. Tensile properties of the material in the two frames are given in Table 1.

Test Methods and Observed Data

Loading Apparatus.—The frames were tested in a 300,000-lb machine having a 21-ft beam which provided an excellent base on which to set the 18-ft models. The load was transferred from the movable head of the testing machine to two fixed load points on the frames by a system of bars and loading beams, as illustrated in Fig. 3, on the square-knee frame.

The horizontal reaction was resisted by a $\frac{3}{4}$-in. round bar extending between the two column bases in all tests. To allow adjustment of the reaction and the span length of the frame, the ends of the tie bar were threaded and fitted with nuts. Rollers under one of the column bases insured that only a negligible amount of friction might affect the horizontal reaction. The tie-bar attachment and rollers are shown in Figs. 3 and 4.

In order to prevent lateral buckling or twisting of the flexible horizontal girder, trussed frames were built up from the testing machine at midspan, and just inside the inner face of each column, as shown in Fig. 3. The frame had a tendency to bear against all of these lateral supports under load; but only at the

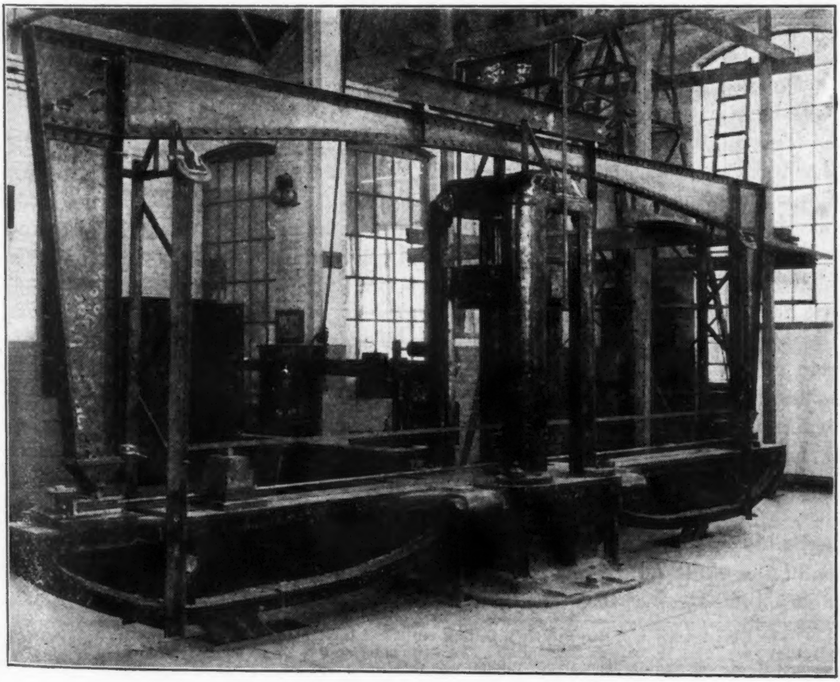


Fig. 3.—Square-Knee Frame in a Testing Machine

center was there any appreciable deflection where frictional resistance to vertical movement might be developed. Comparative tests, with and without the center support, gave practically identical results, so the frictional restraint was considered negligible.

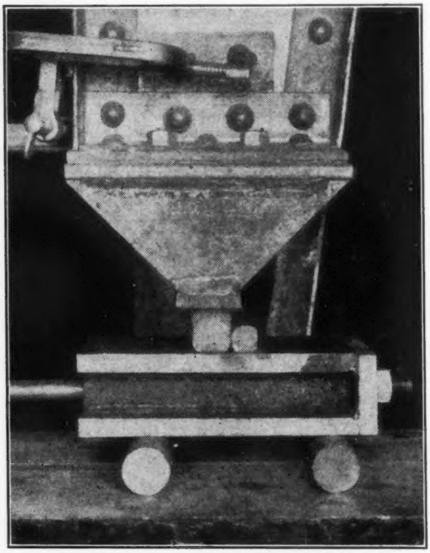


Fig. 4.—Movable Hinged Base

For the most part the rigid frame specimens were tested as two-hinged frames. Therefore, the apparatus shown in Fig. 4 was developed to provide a point bearing for the frame supports and at the same time transmit the horizontal reaction to the tie bar. The bearing area of the pin was always less than 1 in. wide, which was sufficiently small for the assumption of a point support.

It was also considered worth while to investigate how much restraining moment was produced by the base when the frame rested upon flat bases which restrained horizontal movement, but not rotation. To provide this condition, the two sets of apparatus shown in Fig. 5 were developed (the frame proper is indicated by the shaded areas). Since the two gave identical results, Base No. 2 was used in most tests.

Observations and Test Data; Loading.—In all tests the total load was one kip (1,000 lb) for the zero condition and 13 kips for the working condition, giving a working range of 12 kips. Fig. 1 shows the location of the load points. In some of the early tests, the working load was applied in two equal increments, but for the most part it was found convenient to apply the load in only one increment. For the purpose of checking in the latter case, each loading was always repeated.

Stress Distribution at Knee Sections.—To determine the state of stress at each gage point on the web, three strain readings (horizontal, vertical, and inclined at 45°) were observed. Stresses were obtained from the observed strains by the graphical method developed by W. R. Osgood, M. Am. Soc. C. E., and R. G. Sturm,[6] Assoc. M. Am. Soc. C. E. At each flange gage point, only the strains parallel to the longitudinal dimension of the flange were observed, as the transverse stresses in the flange may generally be considered negligible. Wherever possible, flange strains were observed at both heel and toe of the outstanding legs of the angles and on the edge of the web. At all gage points, strains were observed simultaneously on both sides of the frame

[6] "The Determination of Stresses from Strains on Three Intersecting Gage Lines and Its Application to Actual Tests," by William R. Osgood and R. G. Sturm, *Research Paper No. 559, Journal of Research*, National Bureau of Standards, Vol. 10, 1933.

in order to eliminate the effect of lateral bending. These strain observations were made with tensometers having 1-in. gage lengths. With these instruments, stresses could be obtained within an expected accuracy of 300 lb per sq in. The instruments were held in position on the web plate by means of

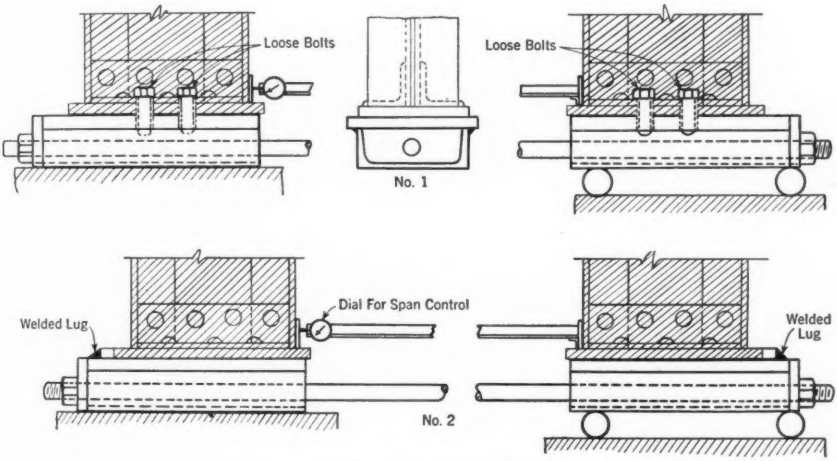


FIG. 5.—FLAT BASE DETAILS

$\frac{1}{8}$-in. bolts and holes tapped in the web. As it was desirable to keep the number of holes at a minimum, only three strain readings were observed at each point. Instead of the check usually obtained by the fourth reading, a repetition was made of each loading and observation. The tensometer attachments for web and flange strain measurements are shown in Figs. 3 and 6.

Three complete sets of stress-distribution data were obtained for the square-knee frame, one set for each knee of the frame at normal span length, and one for the east knee with the supports allowed to move outward $\frac{1}{4}$ in. under load. In general, the results for the two knees were so similar that the data from only one are presented herein. The variation of distribution due to movement of the supports is discussed in another section.

For the curved-knee frame (see Fig. 7), two complete sets of stress-distribution data were observed, of which only one is presented herein, for the reason given in the preceding paragraph. The bases were maintained at the normal distance apart for these tests.

From the strains observed at each gage point on the web of each rigid-frame knee, principal stresses and maximum shearing stresses were determined graphically by the method mentioned previously. On the backs of the flanges longitudinal strains only were observed, from which stresses were computed directly. The values of principal stresses and maximum shearing stresses are indicated by lines of equal stress (contour lines) for the square knee in Fig. 8(a), and for the curved knee in Fig. 8(b). The directions and approximate magni-

tudes of the principal stresses at each of the gage points are also shown in the diagrams.

In the square-knee frame the knee joint may be considered as a rigid beam-and-column connection in which the column extends to the top of the frame. Whether the girder or column extends through the knee depends on the location of the joint in the plate, which in this case was located at the vertical section through the inside corner of the knee. With this conception in mind, normal stresses on sections approximately perpendicular to the axes of the column and the girder were determined from the principal stresses, and are shown in Fig. 9. It will be noted that the neutral axis of the column deviates only slightly from the center of gravity axis. Stresses normal to a plane passing through the inside and outside corners of the knee (section *O–O* in Fig. 9) were also plotted, and the neutral axis with respect to that section was found to be very close to the inside corner. The stress on this plane at the inside corner was computed from stresses parallel to the two flanges at this point, which were extrapolated from observations at the nearest gage points on the compression flanges of the column and girder. The dotted curve on the vertical section in the girder nearest the corner indicates the stress distribution that might have existed on that section if the bearing of the compression flange of the girder upon the column had been properly distributed over the cross section of the flange instead of being concentrated in the outstanding legs of the angles.

In the testing of the curved-knee frame it was observed that the stresses at the toes of the outstanding legs of the curved flange angles were only 40 to 70% of the stresses at corresponding points at the heels of the angles or on

FIG. 6.—TEST SET-UP SHOWING ARRANGEMENT OF TENSOMETERS ON THE SQUARE-KNEE FRAME

the edge of the web. The stresses at the heels of the angles and on the edge of the web agreed very closely. Fig. 10 shows flange stresses observed on three sets of gage lines: On the edge of the web, at the heels of the angles, and at the toes of the angles. This transverse variation of stress in the outstanding legs of flange angles did not exist to any extent on straight flanges whether subjected to tension or compression. The explanation of this phenomenon was found to be a kind of buckling action in which the outstanding legs of the curved flange angles bent away from their center of curvature, thus elongating relative to the edge of the web and relieving some of the compression in the outstanding legs. For the observed differential in stress between the edge of

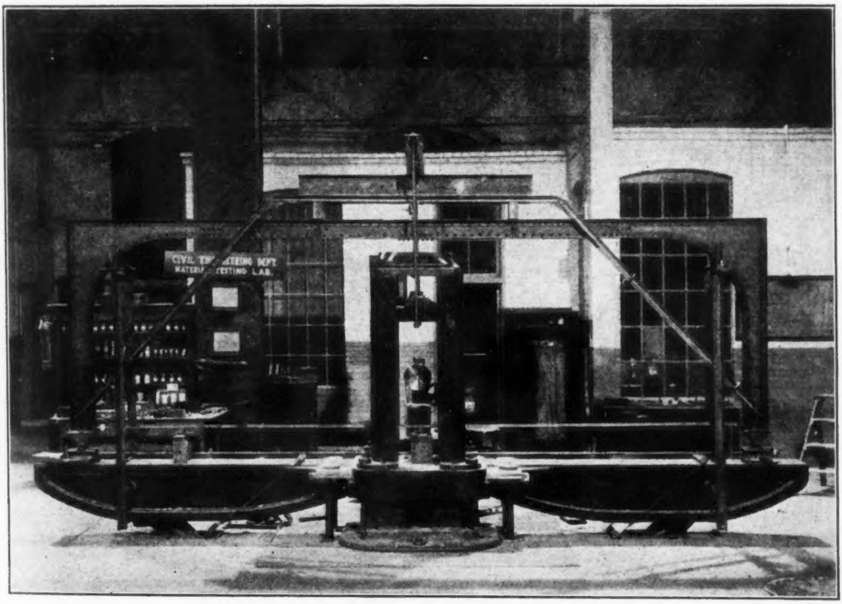


FIG. 7.—CURVED-KNEE FRAME AND DEFLECTION APPARATUS

the web and the toes of the angles to have taken place, the toes of the outstanding legs would have had to deflect about 0.01-in. relative to the edge of the web. The slight transverse bending observed, as measured by a transverse bending stress of about one kip per sq in., was not sufficient, however, to produce this deflection, indicating that some rotation was probably taking place about the rivet line. The stresses on all three sets of gage lines increased approximately in proportion to the load.

Normal stresses on planes radial to the curved flange are shown in Fig. 11. It will be noted that the straight-line distribution of stress of the straight-beam theory does not exist on the sections within the knee, and that the neutral axis is between the center-of-gravity axis and the curved flange. The average

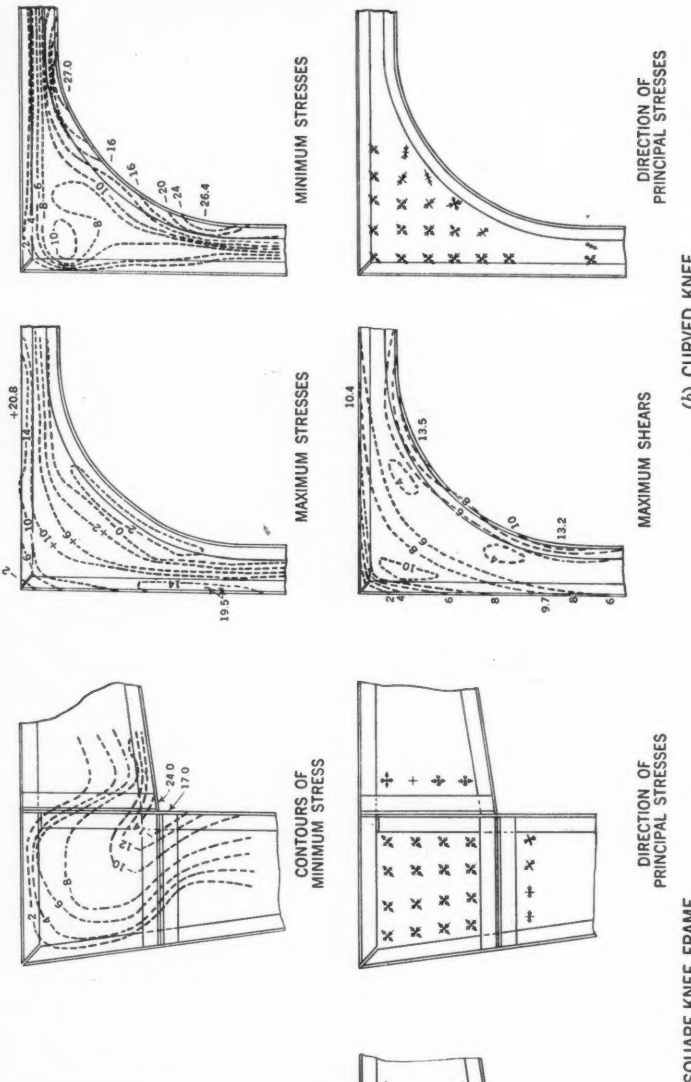


FIG. 8.—OBSERVED PRINCIPAL STRESSES, IN KIPS PER SQUARE INCH

stress values at the back of the curved flange indicated in Fig. 11 were obtained by assuming that the stress was constant across the backs of the angles at each section, that the neutral axis would remain where observed, and that the moment of the compression area about the neutral axis would be the same as observed. From the observed stresses and the position of the neutral axis, the average stress was then computed. The difference between the maximum and average stresses was as much as 25% at some gage points.

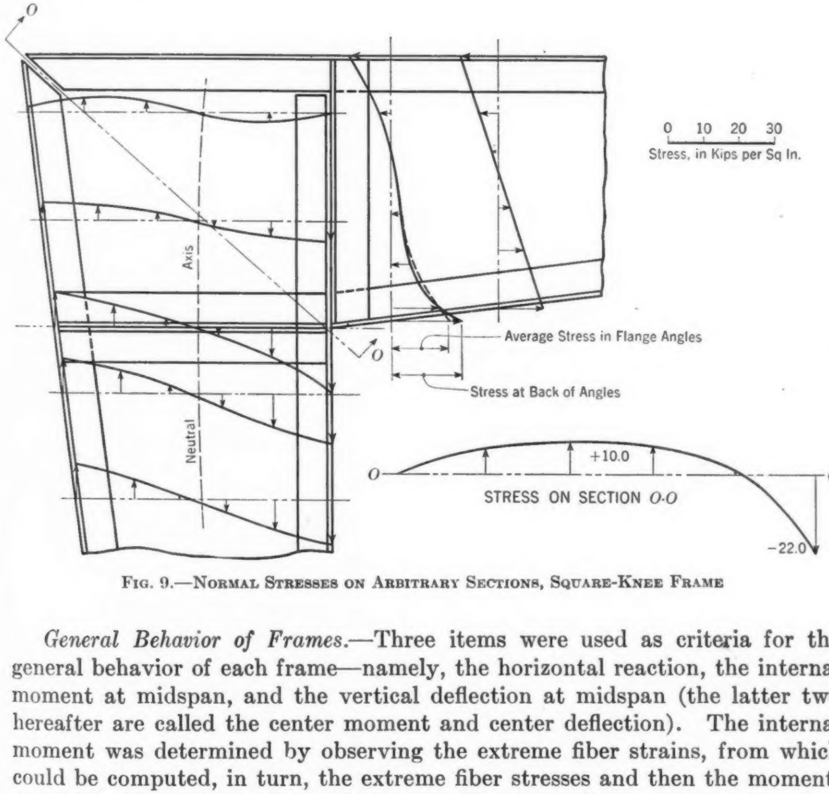


Fig. 9.—Normal Stresses on Arbitrary Sections, Square-Knee Frame

General Behavior of Frames.—Three items were used as criteria for the general behavior of each frame—namely, the horizontal reaction, the internal moment at midspan, and the vertical deflection at midspan (the latter two hereafter are called the center moment and center deflection). The internal moment was determined by observing the extreme fiber strains, from which could be computed, in turn, the extreme fiber stresses and then the moment, assuming that the conventional theory of flexure held true at this location. The horizontal reaction was determined by observing the strain in the tie bar with a 10-in. strain gage and computing the stress and load therefrom. The vertical deflection of the frame at midspan was measured by a 0.001-in. Ames dial between the top flange of the frame and a framework built up from the bases of the frame as shown in Fig. 7.

Load-reaction curves and load-deflection curves for each frame, shown in Fig. 12, illustrate that the frame as a whole behaved as an elastic structure; that is, the observations varied in direct proportion to the load. Individual stress observations were generally made in only one increment; but at several

random points load-stress data were observed, and a straight-line relationship was obtained.

Since only six 1-in. extensometers were available for use (all of these being stationary by nature) many repetitions of loading were necessary to obtain all the desired data. Strain observations on the tie bar were made at regular intervals throughout the course of the testing, and no appreciable change in

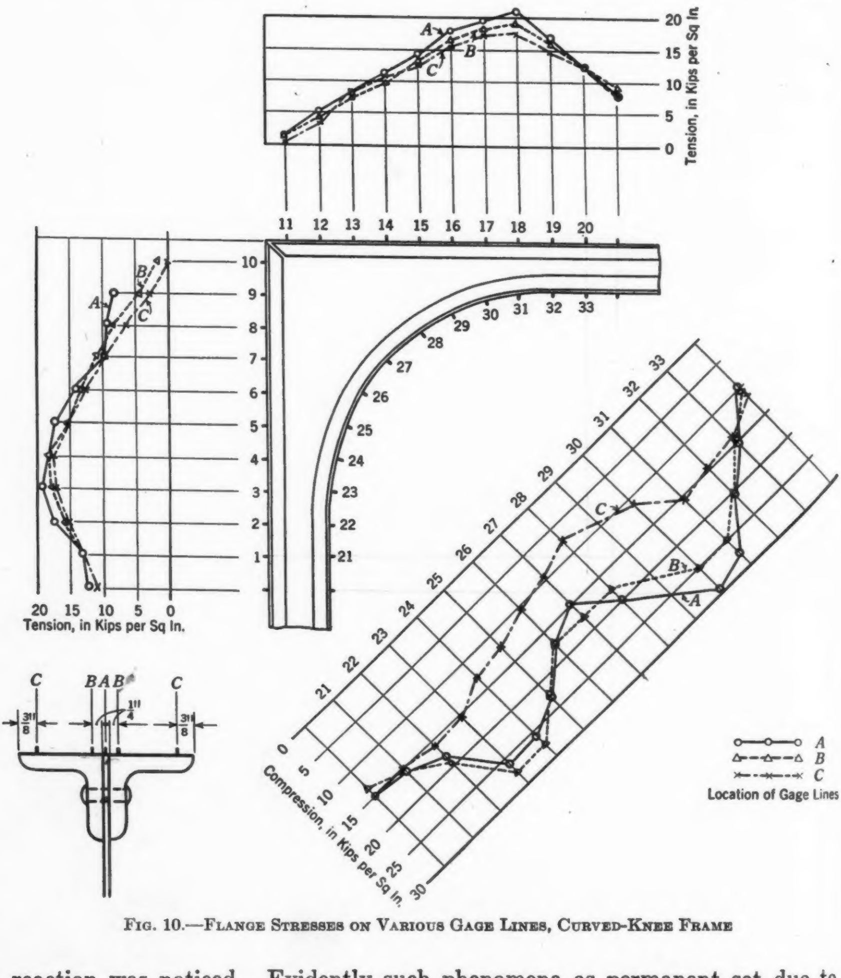


FIG. 10.—FLANGE STRESSES ON VARIOUS GAGE LINES, CURVED-KNEE FRAME

reaction was noticed. Evidently such phenomena as permanent set due to high localized stresses and slippage of rivets, if such existed, had a negligible effect on the frame as a whole. However, a decrease of about 5% in the high localized stresses at the inside corner of the square knee occurred during the course of the testing.

Horizontal Movement of Supports.—In order to simulate one of the most important problems in rigid-frame construction—that of horizontal foundation movement—the bases of the frames were moved inward or outward as the load was being applied. Each frame was studied under five such conditions,

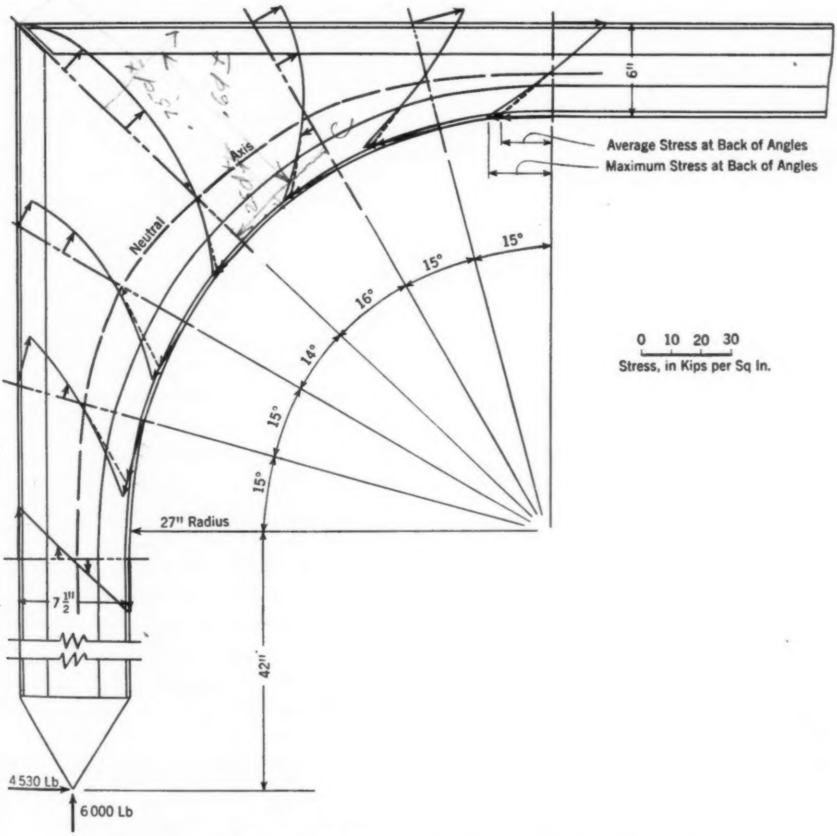


Fig. 11.—Normal Stresses on Radial Sections, Curved-Knee Frame

the span length being varied from the no-load state by the following amounts: $-\frac{1}{2}$ in., $-\frac{1}{4}$ in., 0 in., $+\frac{1}{4}$ in., and $+\frac{1}{2}$ in. A $\frac{1}{4}$-in. movement in the model corresponded to a 1-in. movement in the prototype.

The adjustment of the span length was controlled by a 0.001-in. Ames dial bearing against one column base and fastened to a long light angle clamped at its opposite end to the other column base. Rollers supported the angle along its length. The two ends of this device are shown in Fig. 5. Readings at zero load were always taken at normal span length; that is, the span length

(distance between supports) under no load. As the load was being applied, the span was adjusted by the nuts on the tie bar, so that, at maximum load, the span length was as desired, either normal, or plus or minus a given quantity.

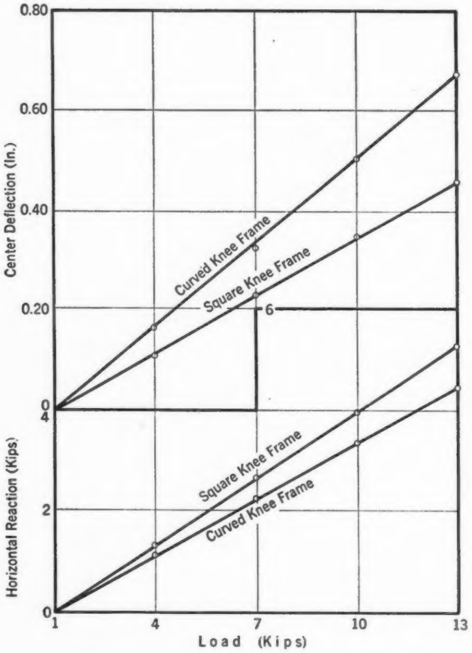


FIG. 12.—OBSERVED LOAD-DEFLECTION AND LOAD-REACTION CURVES FOR NORMAL SPAN

For each test, the horizontal reaction, the center moment, and the center deflection were determined from observations. The values of these quantities for each of the variations of span length are shown plotted with computed values in Fig. 13 for both rigid frames. It is noted that a linear variation was found for each of the items observed.

A complete set of stress-distribution data was observed for one of the knees of the square-knee frame with the supports allowed to move outward $\frac{1}{4}$ in. relative to each other under load. The results obtained were qualitatively similar to those presented in the paper obtained for the normal span condition (Fig. 8(*a*)), the values of the stresses observed being slightly less, as would be expected in view of the fact that the moments and shears acting on the knee are smaller if the supports are allowed to move outward.

Flat Base Tests.—Both frames were tested while supported on the flat plate bases shown in Fig. 5 in order to determine the amount of rotational restraint produced by this type of a base. Internal moments were determined from flange strain measurements at two locations in each frame. Since the horizontal reaction was also observed, it was then possible to compute the location of the point of inflection in the column, if it existed. In both frames, the point of inflection was found to be so close to the base of the column that this type of base was considered to produce, in effect, a hinged support.

Further evidence that the frames resting on the flat plate bases acted as hinged frames is presented in Table 2, which gives ratios of observed to computed horizontal reactions, center moments, and center deflections for both hinged and flat-base tests. All computations were based on the assumption that a hinge existed at the base. It is noted that the two base conditions gave very similar results for both frames.

Relation of Test Data to Analysis

Moment of Inertia.—A test was made to determine the correct moment of inertia for use in computations. With the curved-knee frame inverted in the testing machine, the girder that was of uniform section was tested as a simple beam with two equal and symmetrically situated loads. At a working load

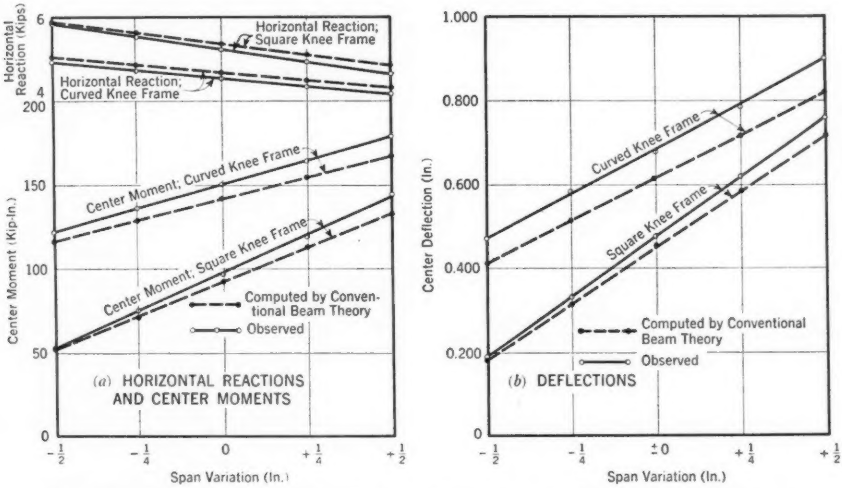


Fig. 13.—Comparison of Observed and Computed Reactions, Moments, and Deflections

for the girder, strains were observed on the top and bottom of the girder. Stresses were computed from the observed strains, and the effective moment of inertia was computed from the extreme fiber stresses, applied loads, and dimensions of the girder.

The effective moment of inertia was slightly less than the gross moment of inertia, and almost exactly equal to that obtained by assuming the total

TABLE 2.—Comparison Between Hinged-Base and Flat-Base Results

$$\left(\text{Ratio} = \frac{\text{Observed}}{\text{Computed}}\right)$$

Type of base	(a) Square-Knee Frame			(b) Curved-Knee Frame		
	Horizontal reaction	Center moment	Center deflection	Horizontal reaction	Center moment	Center deflection
Hinged	0.985	1.06	1.02	0.965	1.07	1.10
Flat..........	0.975	1.08	1.04	0.955	1.09	1.11

volume of the rivet holes to be uniformly distributed along the length of the girder. This average moment of inertia was used in all computations. If the gross moment of inertia had been used, the error would have been about 3% for this girder.

Horizontal Reaction.—In order to compare the test results with computed values, analyses were made for both frames by the Maxwell-Mohr (unit dummy load) method as applied to a two-hinged arch. Since both frames were of non-uniform cross section, the members of each frame were arbitrarily divided into short sections and the necessary integrations were made as algebraic summations. The arbitrary sections are shown in Fig. 1.

The effects of shear and direct stress were included in the computations, since for comparison with test results a greater degree of accuracy than that ordinarily required in design was desired. The equation for the horizontal reaction (H), considering deformations due to bending, shear, and direct stress, can be written:

$$H = \frac{\sum \frac{M_1\, m_h\, \Delta s}{E\, I} + \sum \frac{V_1\, v_h\, \Delta s}{A_w\, G} + \sum \frac{N_1\, n_h\, \Delta s}{A\, E}}{\sum \frac{(m_h)^2\, \Delta s}{E\, I} + \sum \frac{(v_h)^2\, \Delta s}{A_w\, G} + \sum \frac{(n_h)^2\, \Delta s}{A\, E}} \quad \text{..........(1)}$$

in which: M_1, V_1, and N_1 are the actual moment, shear, and thrust in the frame, due to applied loads on a simple frame (that is, assuming no restraint to horizontal movement of the bases); m_h, v_h, and n_h are the moment, shear, and thrust due to a unit horizontal load applied at the supports; Δs is an arbitrary length of section of a frame; E is the modulus of elasticity in tension and compression; G is the shearing modulus of elasticity; I is the moment of inertia; A is the total cross-sectional area; and A_w is the cross-sectional area of the web. Actual evaluation of the different quantities indicated that, for rigid frames of types similar to the two models, the last two terms of the numerator and the last term of the denominator may be neglected for all practical purposes, giving:

$$H = \frac{\sum \frac{M_1\, m_h\, \Delta s}{E\, I}}{\sum \frac{(m_h)^2\, \Delta s}{E\, I} + \sum \frac{(v_h)^2\, \Delta s}{A_w\, G}} \quad \text{................(2)}$$

Neglecting the remaining shear term would introduce an error of about 3% for the square-knee frame and about 1% for the curved-knee frame.

The application of Eq. 2 for H is a routine matter provided the structure behaves in accordance with the assumptions on which the theory is based. Figs. 9 and 11 show, however, that at the knees of both rigid frames straight-line distribution of normal stresses does not exist, and, in all probability, sections that are plane before bending do not necessarily remain plane after bending. As a result, the actual bending deformations $\frac{M\, \Delta s}{E\, I}$ and the approximate shearing deformations $\frac{V\, \Delta s}{A_w\, G}$ in the sections at the knee will differ from those computed by the Maxwell-Mohr theory[7] for straight beams. Therefore, a study of the stress distribution at the knee of each frame was made to determine

[7] "Elementary Treatise on Statically Indeterminate Stresses," by J. I. Parcel and G. A. Maney, Members, Am. Soc. C. E., John Wiley & Sons, Inc., New York, 2d Ed., 1936, Chap. 1.

the treatment of the corner sections that would most nearly approach actual conditions as interpreted from observed data.

The changes in theoretical horizontal reaction due to variations in span length were determined directly from Eq. 2. The numerator of Eq. 2 represents the horizontal deflection of the supports due to the applied loads when the supports are free to move horizontally; that is, the variation in span length for a 100% change in reaction. Since the change in reaction is proportional to the variation in span length for a given loading, the horizontal reactions for the various span lengths were obtained by direct proportion.

Deflections.—Vertical deflections at the center of each frame were computed by the Maxwell-Mohr method, involving the application of a vertical unit load at the point at which the deflection was desired. For convenience, it is recommended that the moments due to the unit load be computed for the determinate frame (supports free to move horizontally), that is:

$$\delta = \sum \frac{M\ m\ \Delta s}{E\ I} \qquad \text{(3)}$$

in which: δ = deflection at any point; M = actual moments in the structure due to applied load; and m = moments due to a unit load applied to the determinate structure at the location of, and in the direction of, the deflection desired.

Corner Section, Square-Knee Frame.—Of primary importance is the degree to which the sections within the knee may influence the determination of the horizontal reaction. Total neglect of knee sections in the computations for horizontal reaction (that is, considering the knee to be infinitely stiff) increased the computed value by about 3%.

An approximation to the actual deformation of the square knee was obtained from the information in Fig. 8(*a*). Since the principal stresses within the knee were approximately circumferential and radial, the knee was divided into circumferential bands, as shown in Fig. 14(*a*). Average radial and circumferential stresses were assigned to the bands in accordance with the observed stress distribution. From the average principal stresses the elongation of each band was computed, which, when plotted together, gave the total angular change between the two internal faces of the square knee, also shown in Fig. 14(*a*). This angular change was practically the same as that computed from the observed shears in the knee (Fig. 14(*b*)), indicating that bending, as ordinarily considered, contributed very little to the deformation of the knee.

The effect of shear in the knee is represented by the analysis presented in Fig. 14(*c*), in which $\sum \frac{V\ \Delta s}{A_w G} = 0.00072$. This practice of treating the column as a free body acted upon by the reactions of the frame and by the flange forces and end shear of the girder (neglecting the moment and thrust carried by the web) is used now by some designers. When the knee is considered to be loaded in this manner, bending about the neutral axis of the column will contribute little to the effective deformation of the knee which was already indicated by test results in Fig. 14(*a*) and Fig. 14(*b*).

Two conventional methods of treating the corner of a square rigid-frame knee for mechanical integration are shown in Fig. 14(d), in which $\sum \frac{M\,\Delta s}{E\,I} = 0.0018$; and Fig. 14($e$), in which $\frac{M\,\Delta s}{E\,I} = 0.0017$. The computed bending deformations at the knee for the two cases were approximately equal and considerably greater

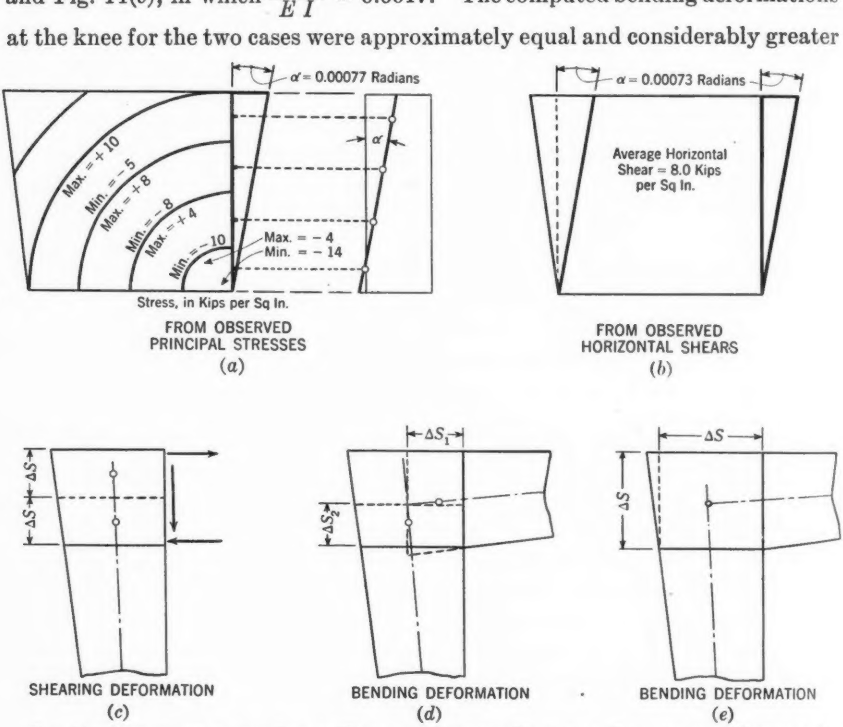


FIG. 14.—DISTORTION OF THE SQUARE KNEE AND POSSIBLE CHOICES OF SECTIONS FOR MECHANICAL INTEGRATION

than the deformation indicated by the analysis shown in Figs. 14(a) and 14(b). If, however, the effect of shear is neglected throughout the frame (as would be done in design), the large bending deformation assigned to the knee tends to offset the absence of shear deformation in the frame as a whole. The results thus obtained differed only slightly from those in which shear was considered. A comparison between the observed horizontal reaction and the values obtained by the several treatments of the knee section is as follows:

Description	Horizontal reaction (lb)
Observed	5,280
Computed by Analysis in Fig. 14(c):	
Shear included	5,340
Shear neglected	5,480
Computed by Analysis in Figs. 14(d) or 14(e):	
Shear included	5,320
Shear neglected	5,360

Computations of all theoretical values used for comparison with observed values were based upon the treatment shown in Fig. 14(*c*), including the effect of shear.

In connection with the rigid-frame investigation conducted at the National Bureau of Standards, a theoretical analysis[3] of a rectangular rigid-frame knee was developed. The analysis involved an application of the theory of elasticity

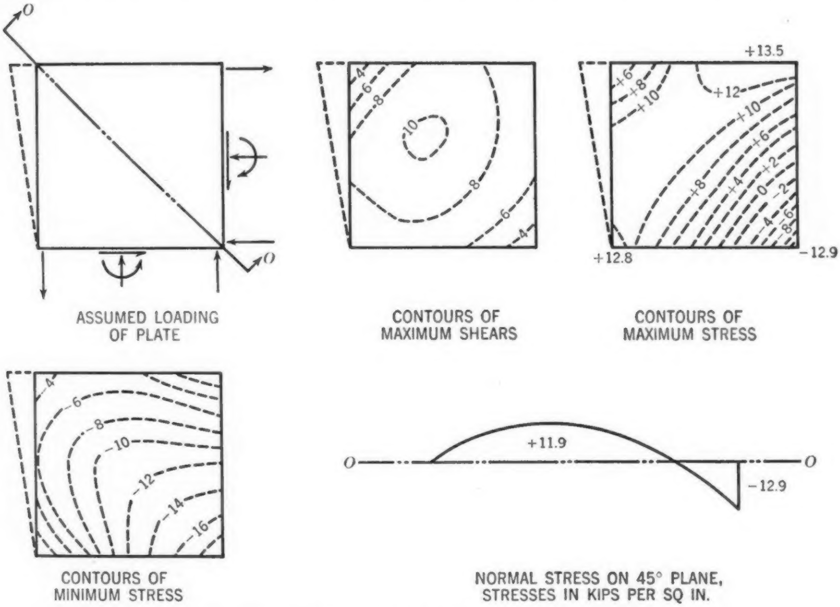


FIG. 15.—ANALYSIS OF RECTANGULAR KNEE BY THE THEORY OF ELASTICITY

to a rectangular plate loaded at the edges with shears, thrusts, and moments in its own plane as shown in Fig. 15. The resulting contours of principal stresses and maximum shears, which were computed by H. D. Hussey, M. Am. Soc. C. E., for a rigid-frame knee of practically identical proportions as the square knee are also given in Fig. 15. By comparison with Fig. 8(*a*) it is noted that the general agreement between theoretical and observed results is very satisfactory.

A comparison of observed and computed extreme fiber stresses in the vicinity of the knee is presented in Fig. 16. Computed stresses are based on the conventional formulas for flexure and direct stress. In plotting the computed tension values it was assumed that the stresses reach a maximum at the section that passes through the inside corner of the knee, and decrease uniformly along the flanges of the knee to zero at the outer corner. The stresses represented by triangular dots on the vertical section passing through the inside corner of the knee were computed on the assumption that the flange angles of the girder transmitted all of the moment and thrust in the girder to the column. This assumption is reasonable if it is noted that the web splice is located at this section.

A simple method for computing normal stresses on the 45° plane through the inside corner of the knee was developed from the test data shown in Fig. 9. In accordance with the stress distribution on the diagonal plane, the neutral axis for this section was assumed to be at one fifth the length of the diagonal from the inside corner, the tension stress area was assumed to be a second-degree parabola, and the compression-stress area a triangle. With these assumptions

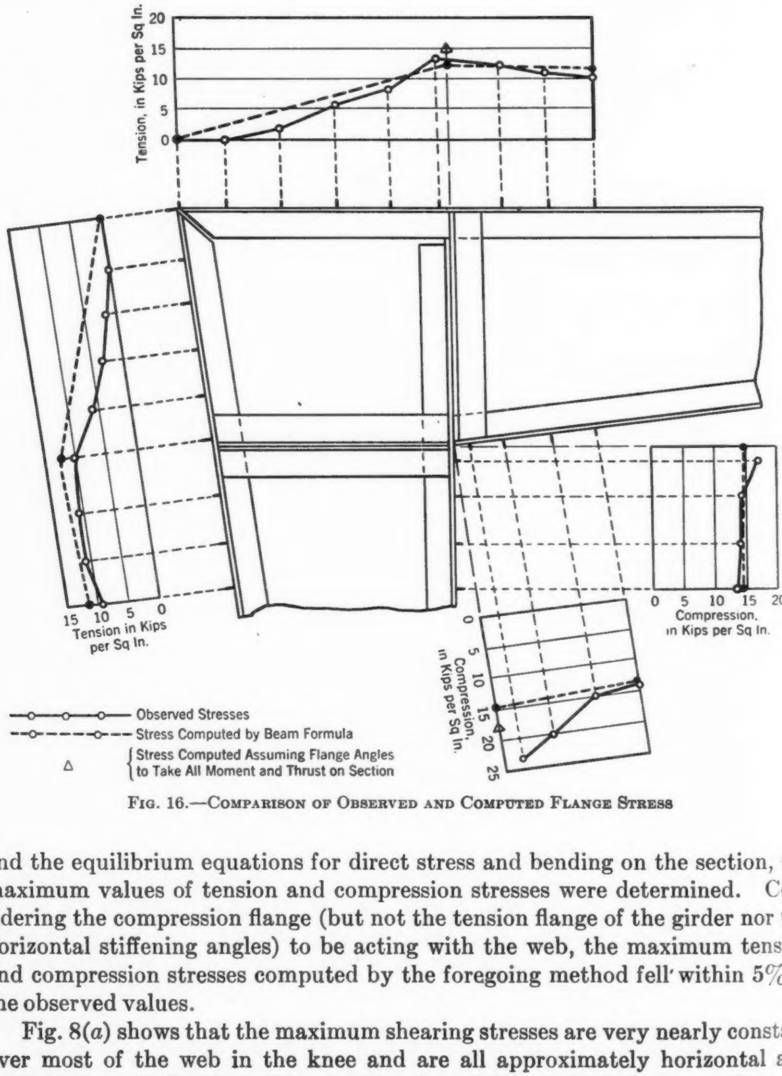
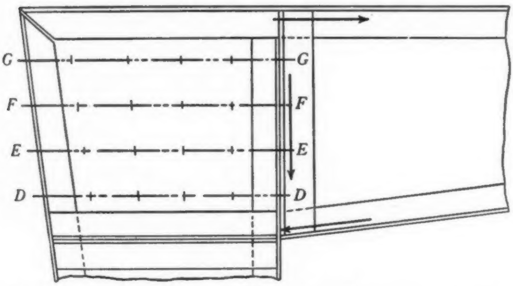


FIG. 16.—COMPARISON OF OBSERVED AND COMPUTED FLANGE STRESS

and the equilibrium equations for direct stress and bending on the section, the maximum values of tension and compression stresses were determined. Considering the compression flange (but not the tension flange of the girder nor the horizontal stiffening angles) to be acting with the web, the maximum tension and compression stresses computed by the foregoing method fell within 5% of the observed values.

Fig. 8(*a*) shows that the maximum shearing stresses are very nearly constant over most of the web in the knee and are all approximately horizontal and vertical, being inclined at 45° to the direction of the principal stresses. Computations show that the average of the horizontal shearing stresses over the entire

knee is 98.4% of the average of the maximum shearing stresses. Thus, a correct determination of the horizontal shearing stresses would be adequate for design purposes. Fig. 17 presents a comparison between the horizontal shearing stresses determined from observations and those computed on the



SECTION	COMPUTED	OBSERVED
D	8 240	8 630
E	8 040	8 120
F	7 860	7 990
G	7 670	7 580

Computed on the Basis of Indicated Loading

FIG. 17.—AVERAGE HORIZONTAL SHEARING STRESSES IN THE SQUARE KNEE, IN POUNDS PER SQUARE INCH

assumption that the web resists a horizontal shear equal to the tension in the top flange of the girder. The girder flanges have been assumed to take all the moment and thrust on the section at the column face as shown in Fig. 17.

Corner Section, Curved-Knee Frame.—In the curved-knee frame the horizontal reaction was 3.5% below the computed value. Average compression-flange stresses at the knee varied from 15% to 50% in excess of stresses computed by the conventional formulas for flexure and direct stress as shown in Fig. 18. Therefore, the bending deformations in sections within the knee were probably greater than those computed by conventional methods—that is, the knee was not as stiff as assumed. To remedy this condition, a simple and arbitrary method for reducing the moments of inertia and section moduli of sections within the knee was developed from the test results. Using these modified values, computations gave a horizontal reaction and compressive stresses within the knee in close agreement with the observed data.

For knees of this type radial sections are the most convenient to use and were employed in all analyses involving mechanical integration. To obtain what will be called the "effective" section modulus (S') for each section, the moment at a given section was divided by the average observed compressive stress at the extreme inner fibers. The ratio of "effective" section modulus to actual section modulus $\left(\frac{S'}{S}\right)$ was determined for each section and plotted against a central angle C, as shown in Fig. 19. The approximately linear relationship between the ratio $\left(\frac{S'}{S}\right)$ and the angle C can be expressed by the linear equation,

$$\frac{S'}{S} = 1 - \tfrac{1}{2}\left(\frac{C}{45}\right) \qquad (4a)$$

that is,

$$S' = S\left(1 - \frac{C}{90}\right) \qquad (4b)$$

Due to the origin of Eq. 4, the compressive stresses based on the "effective" section moduli agreed closely with the average observed compressive stresses. Furthermore, the same principle used in the modification of the section modulus may be applied to the moment of inertia in order to correct for the discrepancy in observed and computed horizontal reactions. The procedure in an actual design, assuming that the stress in the extreme compressive fibers is constant across the width of the flange, would be to compute the "effective" moment of inertia, I', from the relationship:

$$I' = S' c \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

in which c is one half the actual depth of section. In the case of the curved-knee frame, however, the deformation of the knee depended not on the average flange stresses, but on the maximum compressive stresses which occurred at the

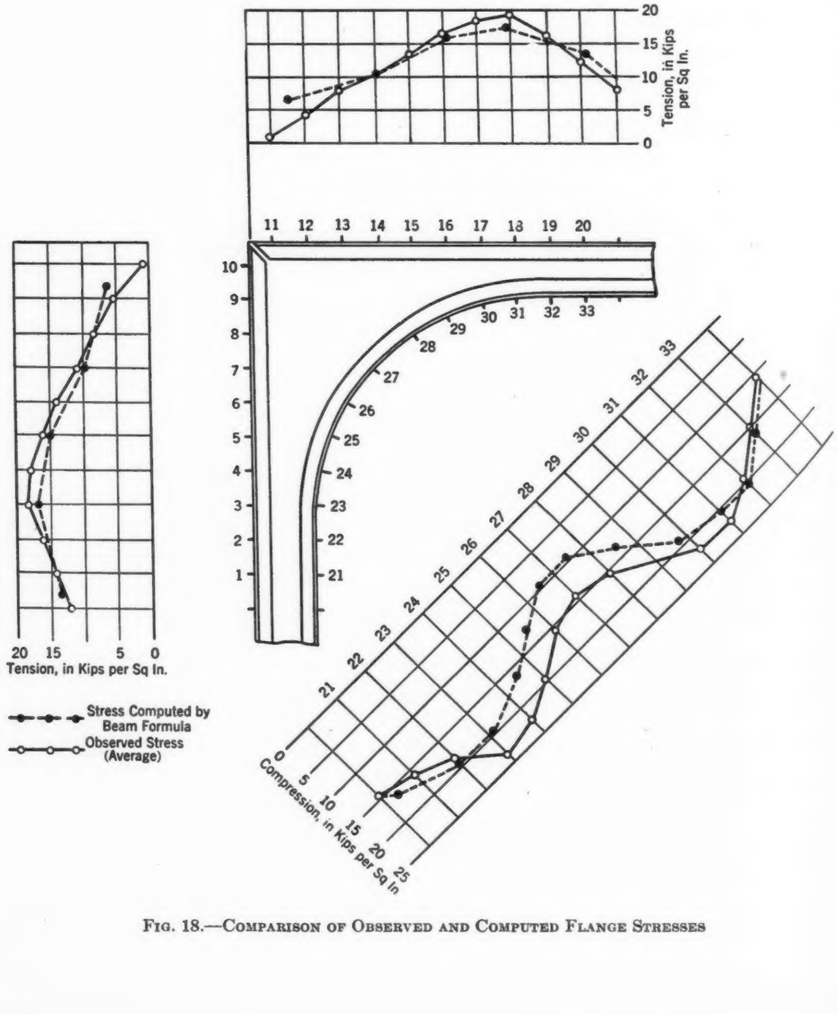


FIG. 18.—COMPARISON OF OBSERVED AND COMPUTED FLANGE STRESSES

edge of the web. By using the "effective" moments of inertia based on the maximum compressive stresses in the web, the value of the theoretical horizontal reaction (neglecting shear) was reduced from 4,690 lb to 4,480 lb, as compared with the observed value of 4,530 lb.

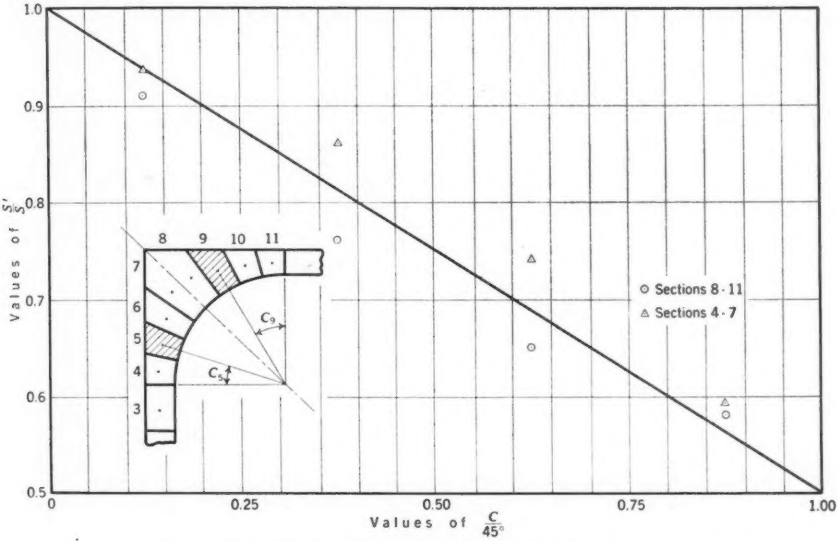


FIG. 19.—GRAPH FOR THE DETERMINATION OF A MODIFIED SECTION MODULUS

Maximum tensile stresses at the knee occurred at the outer extreme fibers of sections near the points of tangency of the curved flange and were in fairly close agreement with values computed by the conventional formulas for flexure and direct stress (Fig. 18). On the radial section through the exterior corner of the knee, however, the maximum tensile stress cannot be computed in this manner. A simple solution for this section was derived from the test results in Fig. 11, identical in principle with the method applied to a corresponding section in the square-knee frame. The assumptions made were that the neutral axis was one quarter the distance from the compression flange to the exterior corner, and that the stress distribution followed a second-degree parabola on the tension side and a triangle on the compression side. Assuming that the tension flange angles did not act with the web, and applying the equations of equilibrium, the maximum observed tension and compression stresses on the section were checked within about 10%. Since this is not likely to be a critical section, the check was considered satisfactory.

In connection with the tests on rigid-frame knees made at the Bureau of Standards, an analysis[4,8] was developed by Mr. Osgood for the knee frame with the curved inner flange. This analysis determines normal and shearing stresses on arbitrary circular-cylindrical sections for members having nonparallel

[8] "A Theory of Flexure for Beams with Nonparallel Extreme Fibers," by W. R. Osgood, *Journal of Applied Mechanics*, September, 1939.

extreme fibers. No consideration is taken of the possibility that the edges of the member may be curved. A comparison of normal stresses computed by this method and observed normal stresses is presented in Fig. 20 for the curved-knee frame. In general, the observed stresses at the extreme fibers are somewhat higher than the computed stresses, although the agreement is considerably

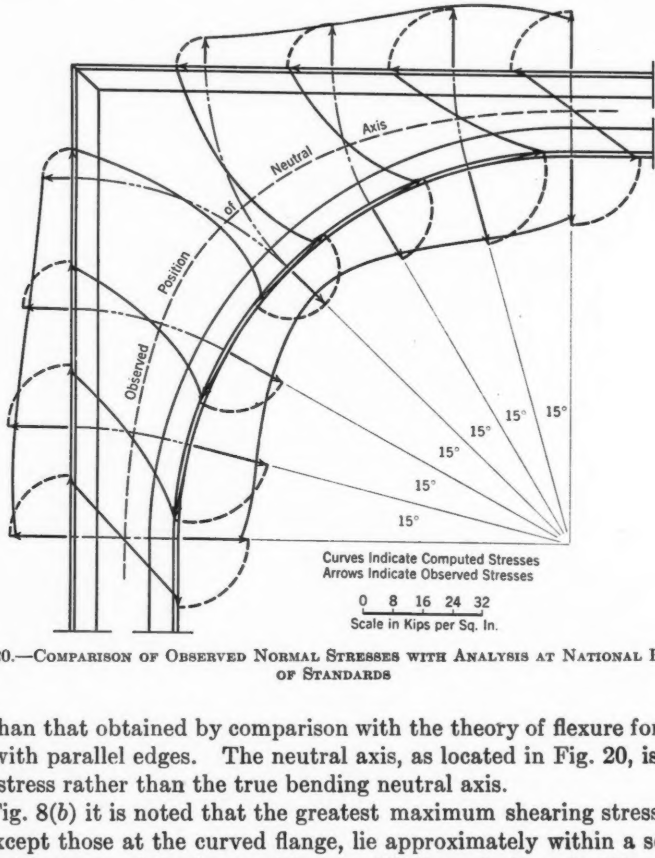


FIG. 20.—COMPARISON OF OBSERVED NORMAL STRESSES WITH ANALYSIS AT NATIONAL BUREAU OF STANDARDS

better than that obtained by comparison with the theory of flexure for straight beams with parallel edges. The neutral axis, as located in Fig. 20, is the axis of zero stress rather than the true bending neutral axis.

In Fig. 8(*b*) it is noted that the greatest maximum shearing stresses in the knee, except those at the curved flange, lie approximately within a square, at the exterior corner of the knee, with two of its boundaries coinciding with the straight flanges of the knee and with its interior corner on the observed neutral axis (see Fig. 11). This square is shown in detail in Fig. 21. From Fig. 8(*b*) it is also noted that the maximum shearing stresses within the square are nearly all horizontal and vertical. The forces acting on this section of the web are illustrated in Fig. 21, the shears introduced by the flange tensions being very great in comparison to the bending loads due to the web. Therefore, the horizontal and vertical shears in the square were considered to be approximately equal to the shear introduced by the flanges. The total stress in the angles that must be unloaded into the web through the rivets along the two exterior sides of the square can be computed by the conventional formulas for flexure and

direct stress. Assuming the square to be isolated from the remainder of the knee and loaded with horizontal and vertical shears equal to the average of the tensions in the two flanges, which were nearly equal, the resulting average shearing unit stress agreed closely with the observed maximum shearing stress. The comparison between observed and computed results is shown in Fig. 21. The high shearing stresses along the curved flange are a direct function of the normal stresses there, and they will not become excessive if the normal stresses do not exceed working limits.

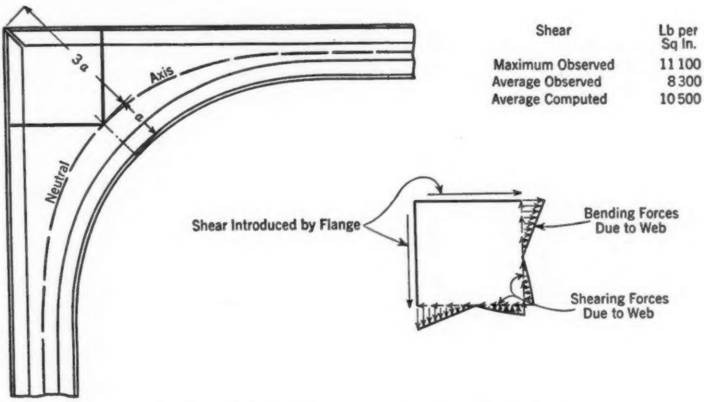


FIG. 21.—SHEARING STRESSES IN THE CURVED KNEE

General Behavior of Frames.—The extent to which the frames, each considered as a whole, behaved in accordance with currently accepted theory is shown in Fig. 13. For both frames the observed horizontal reactions were slightly less than the computed values, varying from 0 to 4% for the square-knee frame, and being constant at about 4% for the curved-knee frame. The observed center moments and deflections were greater than the computed values, varying from 0 to 8% for the square-knee frame, and from 5 to 13% for the curved-knee frame. According to the laws of the statics, observed reactions and center moments should always check each other. In general they agreed very well. Any excessive deformation at the knee sections would be reflected

TABLE 3.—COMPARISON BETWEEN TEST DATA AND COMPUTED VALUES FOR TESTS ON NORMAL SPANS

Description	(a) SQUARE-KNEE FRAME			(b) CURVED-KNEE FRAME		
	Horizontal reaction (lb)	Center moment (kip-in.)	Center deflection (in.)	Horizontal reaction (lb)	Center moment (kip-in.)	Center deflection (in.)
Observed values	5,280	98	0.465	4,530	153	0.678
Computed values	5,340	93	0.455	4,690	143	0.615
Computed from observed horizontal reaction	....	98	0.485	....	154	0.700

in the value of the horizontal reaction obtained, but not in the relation between the observed reaction and observed moments.

By comparing the observed values of horizontal reaction, center moment, and center deflection with theoretical values and with semi-theoretical values obtained by using the observed horizontal reaction, a good indication of the efficiency of the knee joint as well as of the frame as a whole was obtained. Such a comparison is given for each frame in Table 3. All values refer to the normal span condition.

Discussion of Results

Stress Distribution, Square-Knee Frame.—In general, the parts of the frame outside of the knee behaved in accordance with conventional theory. On sections only a few inches away from the boundaries of the knee, straight-line distribution of normal stresses was obtained. However, immediately adjacent to the knee, observed stresses in the compression flanges of both column and girder were considerably greater than stresses computed by the conventional beam formula for flexure and direct stress. Probably two principal causes contributed to this condition. First, the web of the frame was discontinuous through the vertical joint at the knee, which resulted in the flanges of the girder carrying more than their share of the moment and thrust on the section at the column face. This premise is upheld by the normal stress distribution shown in Fig. 9 on the girder section next to the knee. It is noted that on this section the web is understressed and the flange angles overstressed. Second, the presence of shims between the compression flange of the girder and the column caused practically all of the compression in the girder to be transmitted to the column through the outstanding legs of the flange angles. At a point on the girder 2.5 in. from the column face, the outstanding legs were carrying about three fourths of the total load in the compression-flange angles. The average stress in the compression flange agreed closely with the stress computed on the assumption that the girder flanges carry all the moment and thrust in the girder at the knee joint. The high stress noted in the compression flange of the column at the inside corner of the knee was probably caused, in part, by the extreme concentration of bearing at that point. It seems probable that any additional concentration of stress in the column or girder at the knee because of the sharp reentrant angle was of small magnitude.

Whatever concentration of stress does exist is directly dependent upon the bearing condition at the corner. An accurate fit along the entire joint will produce lower stresses in the flanges but higher stresses in the web. Local variations of this nature are of general occurrence in steel structures, where these high localized stresses are usually disregarded as unimportant. Therefore, with regard to concentrations of stress at sections adjacent to the knee, the rigid frame may be treated as any other steel structure. However, a loose fit is to be avoided because of the possible effect on the horizontal reaction and therefore on the stresses at midspan.

Within the knee the stress distribution cannot be determined by any simple theoretical analysis. The application of the theory of elasticity is too complicated and tedious for practical use by designers. Moreover, Figs. 9 and 16

indicate that the critical sections for normal stresses at the knee are the horizontal and vertical sections through the inside corner of the knee, which were discussed in the preceding paragraph.

Shearing stress is apparently quite important in a rigid-frame knee of this type. The unit shearing stress in the web within the knee was about twice the stress just outside the knee, which is ordinarily considered to be the critical section with respect to shear. A study of the maximum shears (Fig. 17) showed that designing for horizontal and vertical shearing stress in the knee, on the basis of an external shear equal to the total tension in the top flange of the girder, was both adequate and correct.

Stress Distribution, Curved-Knee Frame.—As in the case of the square-knee frame, the parts of the frame outside the knee gave results in accord with conventional theory. Within the knee, the stress distribution differed markedly from that obtained by the ordinary beam theory. Throughout the entire curved part of the compression flange at the knee, the observed stresses were much greater than the computed values. Two factors probably contributed to the observed difference. First, the neutral axis has a pronounced curvature at the knee. However, an application of the curved-beam theory could account for only a small part of the differences between observed and computed stresses, particularly on the compression flange. The second and possibly the most important factor is the rapid change of section that occurs at the knee. This latter problem introduces complexities that may be solved reasonably only by a highly theoretical analysis, or by an arbitrary procedure based on test results. The arbitrary procedure presented in the section on analysis gave good agreement with the observed results, but it made no differentiation between the separate effects of the two aforementioned factors. Such a differentiation would be necessary if the method of analysis were to be extended to knees having decidedly different degrees of curvature or different proportions of flange area to web area.

The stress distribution was complicated by a stress relief in the outstanding legs of the curved flange angles. The radial component of compression due to the curvature of the flange caused the outstanding legs to deflect away from the center of curvature and thus elongate relative to the edge of the web. However, if there had been a cover plate on the backs of the curved flange angles, it is possible that no appreciable radial displacement of the outstanding legs could have occurred. Not only would the outstanding legs have been reinforced by an additional thickness of metal, but any rotation of the individual angles about the rivet line would have been effectively reduced. In the section on analysis it was assumed that cover plates would be present. Consequently the effect of stress relief in the outstanding legs was eliminated by using an average extreme fiber stress, although this phenomenon is too important to be overlooked in design.

The analysis developed by Mr. Osgood was applied to the curved-knee frame, and it gave fairly good agreement with the observed results, although the observed extreme fiber stresses along the curved flange were somewhat higher than those computed by this method. In the Bureau of Standards' tests on a rigid-frame knee of similar shape and proportions, the observed

extreme fiber stresses along the curved flange were slightly lower than the computed stresses. This apparent discrepancy might have been caused, in part, by the presence of the transverse variation of stress across the outstanding legs of the curved flange angles. In the tests described herein, strains were measured on several gage lines (Fig. 10) and an attempt was made to determine a properly weighted average extreme fiber stress. To the best knowledge of the writers, in the Bureau of Standards' tests, strains were measured on two symmetrically situated gage lines, each about midway out on the outstanding leg of a flange angle. The difference in the manner of observation would cause the Lehigh results to be higher relatively than the Bureau of Standards' results, and may afford the explanation for the apparent disagreement cited. Since this method of analysis did not take into consideration the curvature of the frame at the knee, it is not without reason that the observed normal stresses in the curved flange might be slightly greater than the computed values. If the effect of the curvature of the neutral axis were taken into account, it is probable that an even better agreement with the observed compressive stresses could be obtained.

General Behavior.—In both frames the observed horizontal reactions were slightly lower than the computed values, 1.5% for the square-knee frame and 3.5% for the curved-knee frame. These relations refer to the normal span condition. Although this degree of accuracy appears to be satisfactory, the discrepancy is on the dangerous side. A decrease in the horizontal reaction will produce an increase of much greater magnitude in the center moment. A greater degree of accuracy in the computations was obtained for the curved-knee frame by arbitrarily reducing the moments of inertia within the knee in accordance with observed stresses.

In general, observed center moments and deflections agreed very well with corresponding values computed from the observed horizontal reactions. Therefore, the accuracy with which stresses and deflections can be computed is merely a reflection of the accuracy of the computations for the horizontal reaction. In other words, the knee of a two-hinged rigid frame affects the frame as a whole only in so far as it affects the horizontal reaction.

The observed normal span deflections of 0.46 in. and 0.67 in. for the square-knee and curved-knee frames, respectively, seem large when it is considered that the deflections in the prototypes are four times as great. In this connection, reference is made to deflection tests,[9] reported by R. M. Hodges, M. Am. Soc. C. E., on three structural-steel rigid-frame bridges in Westchester County, New York. In these tests, the floor systems and cutoff walls, which were not included in the deflection calculations, evidently acted with the frame girders to a great extent. The observed deflections in some cases were only one eighth of the computed values.

Foundation Slippage.—The importance of preventing horizontal movement of the supports of rigid frames is clearly illustrated by Fig. 13. For an increase in span length of $\frac{1}{4}$ in. (that is, each support deflecting outward $\frac{1}{8}$ in.) the horizontal reaction for the square-knee frame decreased 7% and the center

[9] "Deflection Tests Show Rigidity of Steel Rigid-Frame Bridges," by R. M. Hodges, *Engineering News-Record*, September 3, 1931.

moment increased 20%. Corresponding variations were not quite as great for the curved-knee frame because of its greater flexibility.

The accuracy of computations for the effect of horizontal movement of the supports upon the frame as a whole depends chiefly on the accuracy with which the horizontal reaction is determined. Changes in horizontal reaction for known variations of span length did not check the computed values exactly (the computed and observed curves in Fig. 13 are not exactly parallel), but the agreement was satisfactory for all practical purposes.

Flat Base Tests.—It is of interest to note that practically no rotational restraint was developed at the supports by allowing the base plates of the rigid-frame models to rest upon flat plates which resisted only horizontal movement. In fact, the horizontal reactions were less than the values computed, assuming the supports to be hinged, whereas the presence of any reaction moment should have the effect of increasing the reaction. This might have been due to a slight misalinement of the frames. If the bases were not truly horizontal, the reaction might easily have been located inside the centroid of the base. Such a condition would have produced a reduced horizontal reaction. However, the test results shown in Table 2 definitely indicate that, unless specific provisions are made to prevent rotation at the base of a rigid frame, the frame will act as a two-hinged structure.

Recommendations for Design

For most practical purposes, horizontal reactions in two-hinged rigid frames of the types tested in this investigation may be computed satisfactorily by any theoretically sound method of analysis. Fig. 13 presents a criterion of the accuracy that can be expected.

If greater accuracy is desired for the curved-knee frame, the following recommendations are made: Arbitrarily reduce the moments of inertia of sections within the knee by means of the graph in Fig. 19. Shear is of secondary importance and need not be considered when radial sections at the knee are used.

Design.—Concerning the design of the knee in each frame, the following recommendations are made:

For a square knee,

1. The horizontal and vertical sections through the inside corner of the knee are critical sections with respect to normal stresses. Apply the usual formula for flexure and direct stress to the horizontal section. On the vertical section, assume that the flange angles carry all the moment and thrust in the girder, and apply the same formula.

2. Design the web to take a total horizontal shear equal to the tension in the top flange of the girder computed in step 1.

3. If the web plate is thin in comparison to that in the test specimen, it may be well to investigate the tension in the web on the 45° plane through the inside corner. For this section, the neutral axis will be located at about one fifth of the length of the diagonal from the inside corner, and the stress distribution may be considered to take the form of a second-degree parabola on the tension side, and of a triangle on the compression side. Maximum stresses

may then be computed from the equations of equilibrium for bending and direct stress on the section.

For a curved knee,

1. Critical sections for direct stress at the knee occur within 15° from the points of tangency of the curved flange. Maximum compression stresses may be determined by use of conventional formulas for flexure and direct stress in straight beams if the section moduli are reduced in accordance with Fig. 19. Maximum tension stresses may be determined in the same manner, but using nominal section moduli. Flange stresses may also be computed satisfactorily by the Osgood theory.[8]

2. The web should be designed for shear on the basis of the arbitrary method illustrated in Fig. 21. Shears outside the square will be smaller than those computed for the square.

3. If the web plate is thin in comparison to that in the test specimen, the 45° plane through the outside corner should be investigated, using the same procedure that was recommended for a corresponding section in the square knee. However, in the curved knee the neutral axis is located at a distance from the compression flange equal to about one fourth the entire diagonal distance.

Summary

The important findings in this investigation may be summarized as follows:

(*a*) The square knee did not act as a continuous homogeneous corner. It may be analyzed and designed as a rigid girder and column connection, in which the reactions of the girder upon the column consist chiefly of a top-flange tension, a bottom-flange compression, and a vertical shear, neglecting the moment and thrust carried by the web at the splice.

(*b*) Stress concentrations at the sharp reentrant angle of the square knee were due principally to imperfect bearing rather than to any inherent property of rigid-frame knees.

(*c*) Normal stresses upon radial sections of the curved knee did not exhibit a linear relationship. The neutral axis lay between the centroidal axis and the compression flange. On the radial section through the exterior corner the neutral axis was about one fourth the distance from the compression flange to the exterior corner.

(*d*) Maximum stresses in the curved knee occurred on sections just inside the points of tangency of the curved flange. Average extreme fiber stresses in the curved flange were from 15 to 50% higher than stresses computed by the conventional formulas for flexure and direct stress in straight beams.

(*e*) A simple method of reducing section moduli and moments of inertia within the curved knee produced close agreement between observed and computed values for compressive stresses in the curved flange and also for the horizontal reaction.

(*f*) The outstanding legs of the curved-flange angles deflected away from the center of curvature under the radial component of compression introduced by the curvature. As a result, the stresses observed near the toes of the

outstanding legs at various points along the curved flange were only from 40 to 70% of the stresses at corresponding points on the heels of the angles and on the edge of the web. Maximum stresses on the edge of the web were as much as 25% greater than the average values used for comparing test results with computations.

(*g*) Conventional methods of rigid-frame analysis gave horizontal reactions slightly greater than those observed; $1\frac{1}{2}$% for the square-knee frame and $3\frac{1}{2}$% for the curved-knee frame at normal span.

(*h*) Conventional methods gave center moments and deflections as much as 10% less than observed values for both frames.

(*i*) Center moments and deflections generally agreed closely with values computed from observed reactions and applied loads.

(*j*) Changes in horizontal reaction, center moment, and center deflection due to horizontal movement of supports also checked theoretical values fairly closely.

(*k*) No appreciable rotational restraint was developed by setting the base plates of the rigid frames on flat plates which resisted horizontal movement of the supports, but not rotation. Rigid frames in which no special precaution for preventing base rotation is taken, therefore, will act as two-hinged frames.

(*l*) The results of this investigation agreed very well in general with the results obtained at the National Bureau of Standards and with the English tests reported by J. J. Leeming and S. C. Redshaw[10] in 1939.

Acknowledgments

The program for this investigation was prepared by the Technical Research Committee of the Institute, consisting of Jonathan Jones, M. Am. Soc. C. E., chairman; O. E. Hovey, Hon. M. Am. Soc. C. E.; the late H. G. Balcom, the late Aubrey Weymouth, and F. H. Frankland, H. D. Hussey, and J. R. Lambert, Members, Am. Soc. C. E. Acknowledgment is due to the members of the committee for their active interest in the work and their advice and guidance; to E. L. Durkee, M. Am. Soc. C. E., G. L. Gray, Jun. Am. Soc. C. E., and W. B. McLean Assoc. M. Am. Soc. C. E., of Fabricated Steel Construction, Bethlehem Steel Company, for the design of the square-knee model and for their valuable assistance in the theoretical analysis of the frames; to Mr. Hussey for the design of the curved-knee model; and to the members of the laboratory staff for their assistance in testing the models and the preparation of the report.

[10] "The Testing of Two Portal Frame Girders," by J. J. Leeming and S. C. Redshaw, *Structural Engineer*, No. 17, 1939, pp. 124–133 and 156.

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AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

RELIABILITY OF STATION-YEAR RAINFALL-FREQUENCY DETERMINATIONS

By Katharine Clarke-Hafstad[1]

Synopsis

The accuracy of rainfall frequency values should be considered carefully in the design of flood and erosion control structures. This paper is concerned with the factors affecting the accuracy of rainfall frequency determinations. A method involving a statistical test for persistence is suggested for estimating the reliability of frequencies calculated by the station-year method.

Introduction

Determinations of the frequencies with which rainstorms of certain extent and intensity occur, together with measures of the reliability of these determinations, are essential to the solution of problems of flood control and of soil and water conservation. In the construction of soil-conserving devices such as terraces, check dams, and contour furrows, designed to decrease runoff and soil loss, it is necessary to know the range of stresses to which rainfall will subject these various devices, and the frequency with which these stresses will occur.[2] The size of dam or other control structure which it is economical to build on a given watershed depends chiefly upon the size and frequency of floods in that watershed. In order to predict the frequencies of various amounts of runoff, a knowledge of the frequencies of rainstorms of various sizes is required.

In the present state of the knowledge of the physical controls of weather and climate, the only feasible method for predicting the probable frequencies of certain amounts of rainfall is one based on a statistical study of precipitation history. The record of past precipitation, available in the form of measurements by rain gages, however, provides an extremely inadequate statistical basis for making predictions of future precipitation. Rain gages set up to

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **March 15, 1941.**

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[2] "Climatic Research in the Soil Conservation Service," by C. Warren Thornthwaite, Benjamin Holzman, and David I. Blumenstock, *Monthly Weather Review*, U. S. Weather Bureau, Vol. 66, 1938, pp. 351–368.

sample rainstorms have been very widely spaced, an insufficient number have been of the continuously recording type, and relatively few years of record have been obtained. As a result of the inadequate sampling, very little quantitative and reliable information is available concerning the areal extent of rainstorms, their duration, intensities, and frequencies.

Primarily to mitigate the limitation of the paucity of years of record to be used in calculating probable rainfall intensities and frequencies, the station-year method was devised. This method makes possible the determination of the frequency of those very intense rainfalls which occur so infrequently that only a few observations of their occurrence will be found among the records of a group of stations. It is particularly useful for determining the average number of occurrences of a given amount in a certain length of time when this period is longer than the actual number of years of rainfall record.

Although first applied to records of stream gages,[3] the station-year method was used as early as 1917 by the engineers of the Miami Conservancy District in their elaborate study of storm rainfall of Eastern United States.[4] Their technique has been used in its general outline by all subsequent investigators who have desired to determine for any locality the maximum rainfall to be expected once on the average in any given length of time.

The method consists essentially of collecting for an area all the rainfall records of whatever length, and treating the sum of all the records for all stations (called the "station-year record") as if it were a single record for the midpoint of the area under consideration. A certain time interval is then selected, which may be anything from minutes to months, depending upon the information desired and the data available. All rainfall quantities for this time interval appearing in the aggregate record are then arranged in order of magnitude. The highest amount is considered to be the maximum to be expected in the area, on the average, once in the period of years represented by the station-year record; the second highest amount may be expected once in one half the aggregate record, and so on for smaller quantities. The combined experiences of all stations in the area are assumed "* * * to give a weighted average which may be regarded as the probable average experience for any one point within that area."[5]

The maximum rainfall to be expected with a certain frequency is termed the "pluvial index" for an area. These pluvial indexes are plotted on charts and iso-pluvial lines drawn by interpolating between the indexes. In drawing these lines the assumption is made that the pluvial index refers to the midpoint of the area for which it was determined rather than to any point. This may or may not be a correct assumption, depending (as will be shown) upon the distribution of stations and the homogeneity of rainfall characteristics of the area.

Analysis of the station-year method shows that by this technique one is determining, in effect, the reciprocal of the average number of occurrences of

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[3] "Flood Flows," by W. E. Fuller, M. Am. Soc. C. E., *Transactions*, Am. Soc. C. E., Vol. LXXVII, December, 1914, pp. 564–617.

[4] "Storm Rainfall of Eastern United States," Miami Conservancy District, Eng. Staff, *Technical Reports*, Pt. 5, Revised Edition, 1936, Dayton, Ohio.

[5] *Loc. cit.*, p. 63.

a given amount of rainfall, per station per year. If 500 station years of record yield 5 occurrences in excess of a certain amount, the average number of occurrences per station per year will be $\frac{1}{100}$. This is equivalent to an average frequency of 100 years per occurrence, which is the same result that would be obtained by taking the fifth highest amount in a station-year record of 500 years.

It should be noted that the station-year method is a simple statistical procedure for arriving at an approximate answer to a problem which it is hoped may some day be solved more satisfactorily either from a consideration of the meteorological processes producing rainstorms, or through the accumulation of more detailed data and a study of rainstorm morphology. Ideally, one would like to know the frequency of rainstorms of certain area, intensity, and duration—not simply the frequency of a certain amount of rainfall for a point in an area. Aside from the restriction due to length of record, the chief limitation in the usefulness of the station-year method is its failure to indicate the size of area which may be expected to receive the given amount of precipitation. At present very little information is available on the intensity-area relations of storms, and this is due to the fact that, until recently, recording rain gages have been so widely spaced that storm areas could not be determined with any degree of accuracy. This deficiency of gages is gradually being remedied.

In certain areas of Eastern United States many more rain gages have been installed within recent years. In the watersheds of the Susquehanna River and the Ohio River above Pittsburgh, Pa., in September, 1940, there were about 100 automatic recording gages in operation, approximately 1 for every 400 sq miles. No single agency assumes responsibility for maintaining all these gages. The Pennsylvania Water and Power Resources Board; the United States Geological Survey, Weather Bureau, Forest Service, and Soil Conservation Service; local power companies; and other agencies have each placed their gages in locations most advantageous to themselves. The collection and compilation of the data from all these gages, however, will contribute fundamental information on rainstorm frequency and intensity that will be extremely valuable to all agencies concerned with the precipitation on this watershed. Similarly, in the Tennessee Valley,[6] a region of approximately 40,000 sq miles, there were, in July, 1940, about 460 rain gages maintained by various agencies, of which approximately 70 were of the recording type. Most of these gages have been in operation 5 years or more. The maintenance of such networks as these, together with the very dense net of automatic gages, 500 in 8,000 sq miles, established by the Soil Conservation Service on the Muskingum Watershed in Ohio, will make possible the determination of the frequencies of rainstorms of given area and intensity.[7] The reliability of the frequency data from these networks of stations will increase rapidly as the years of record increase.

[6] "Precipitation in Tennessee River Basin," Tennessee Valley Authority, Hydraulic Data Div., *Annual*, 1938.

[7] "Microclimatic Studies in Oklahoma and Ohio," by C. Warren Thornthwaite, *Science*, Vol. 86, 1937, pp. 100–101.

Factors Affecting the Accuracy of Frequency Determinations

The simple computation of average frequency by the station-year method requires no information regarding the relation of rainfall amount to the associated storm, but a knowledge of the characteristics of rainstorms is necessary for the correct interpretation of the results.

If there is a closely spaced, well-distributed net of recording stations in an area, and a large number of years of record—not station years, but actual years of time—the average frequency of a specified amount of rainfall computed by the station-year method is a reliably representative one for the midpoint of that area for which it was determined. The relationship between the length of record and the accuracy of the average frequency is the chief concern of this paper. The arguments will be based on the fact that the statistical theory of probability makes it possible to indicate, by a percentage of error in the average, how the reliability of a frequency value is related to the length of independent record.[8]

Number and Distribution of Stations.—Some of the ways in which the accuracy of the average frequency will be affected by the number and distribution of recording stations will be briefly mentioned before treating the effect of the length of record, although it should be emphasized that much more data and study on intensity-area relationships are required before any improvement can be made in the accuracy of frequency determinations.

C. Warren Thornthwaite[9] has shown that with a widely spaced net of stations it is highly improbable that the maximum rainfall occurring in a storm will be recorded at any station. This is illustrated by the fact that no gage recorded the rainstorm which produced a flood on the Republican River in May, 1935. If maximum quantities are not recorded for all rainstorms, the computed maxima to be expected with a certain frequency certainly will be too low.

An average frequency value for an area calculated from the data for several stations will be representative for any point in the area only if the rainfall characteristics are fairly uniform throughout the area. Although no area in the United States as large as a quadrangle of 2° of latitude and longitude is homogeneous with respect to its rainfall characteristics, the engineers of the Miami Conservancy District stated that the station-year method, as used in their study, presupposes

> "* * * that the rainfall characteristics, especially with regard to high rates of precipitation, are essentially uniform at all points within the area of a 2-degree quadrangle of the earth's surface."[10]

They recognize, of course, that this supposition is not strictly true because on the charts of pluvial indexes[11] they have drawn iso-pluvial lines which indicate transitions from one pluvial index to another across a 2° quadrangle.

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[8] "A Statistical Method for Estimating the Reliability of a Station-Year Rainfall-Record," by Katharine Clarke-Hafstad, *Transactions*, National Research Council, Am. Geophysical Union, Vol. L, 1938, pp. 526–529.

[9] "The Reliability of Rainfall Intensity-Frequency Determinations," by C. Warren Thornthwaite, *loc. cit.*, Vol. 2, 1937, pp. 476–484.

[10] "Storm Rainfall of Eastern United States," Miami Conservancy District, Eng. Staff, *Technical Reports*, Pt. 5, Revised Edition, 1936, Dayton, Ohio, p. 67.

[11] *Loc. cit.*, Figs. 13–36, pp. 69–80.

Since it is obvious that no 2° quadrangle of the earth's surface will have uniform characteristics of rainfall, other methods of selecting areas should be considered. More representative frequency-intensity values could probably be obtained by calculating the averages for overlapping circular areas whose radii would be equivalent in length to one side of a 1° quadrangle and whose centers would be corners of 1° quadrangles.

More accurate values might be obtained, also, by selecting the homogeneous areas on the basis of regions having the same pluvial indexes as shown by the iso-pluvial lines on the Miami charts. Prof. Eugene L. Grant,[12] M. Am. Soc. C. E., has proposed the use of the χ^2-test to delimit a homogeneous area whose pluvial index would be representative of any point in the area.

In regions with mountains or other features that result in great differences in rainfall characteristics in short distances, it is practically impossible to obtain average frequencies that have any real significance for large areas. A very dense and uniformly spaced net of rain gages would be necessary to record all the areal differences in order that the average value would represent the area as a whole, and even this value would depart greatly from the true value for certain parts of the area. Average frequencies for mountainous parts of the United States are especially unreliable because in these regions the rain gages are insufficient in number and are located chiefly in the valleys.

An irregularity of station spacing will seriously affect the values of average frequency in areas with non-uniform rainfall characteristics. If there are 2 gages, in any area, several miles apart, one may record 3 occurrences of a certain amount per year, and the other gage 6 such occurrences; but if there had been a third gage so close to the gage recording 6 occurrences that it gave an identical record, the average for the area would be distorted in favor of the higher number of occurrences. Since there is probably no area with absolutely uniform rainfall, an irregularity of station spacing will affect all frequency determinations to some extent.

The accuracy of frequency values depends to a very great degree upon the adequacy of the sampling of rainstorms. It is logical to question whether, from a given net of recording stations, fair and adequate samples of the various rainfall quantities are being obtained. This depends not only upon the number and distribution of the stations, but upon the relation of this distribution to the distribution of precipitation.

In general, rainstorms are of two types: (*a*) Those of wide extent (sometimes hundreds of thousands of square miles, and characterized by a small range of intensities, which results in a rather uniform rainfall pattern); and (*b*) those limited in area to a few hundred or thousand square miles (characterized by a wide range of intensities resulting in a very spotty distribution pattern). Gradations from one type to the other occur, and combinations of the two types result in very complex patterns.

The fundamental distinction between these two patterns is based on the type of physical process that causes the precipitation. The spotty rainfall pattern results primarily from rain originating through a convective process; the widespread homogeneous pattern almost always results from frontal

[12] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 384.

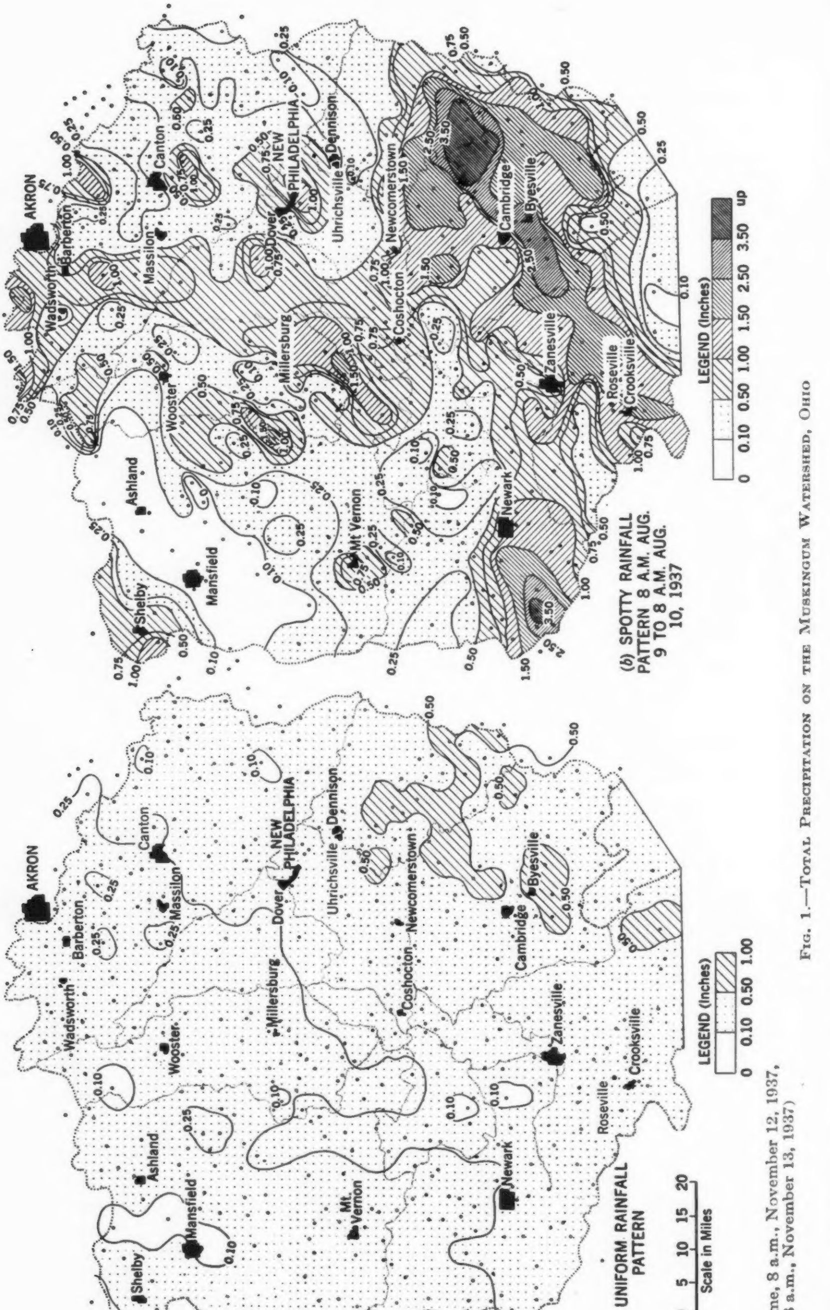

FIG. 1.—TOTAL PRECIPITATION ON THE MUSKINGUM WATERSHED, OHIO

(Time, 8 a.m., November 12, 1937, to 8 a.m., November 13, 1937)

activity whereby moist, less dense air is cooled adiabatically by ascent over colder, more dense air to the point where the moisture condenses.

The two general types of precipitation patterns are illustrated by the storms of November 12–13 and August 9–10, 1937, which occurred in the Muskingum Watershed in Ohio (Fig. 1). Rainfall measurements from the 500 automatic recording gages installed by the Soil Conservation Service on the 8,000 sq miles of this watershed provide the data for the maps of these storms. The November storm illustrates the uniform pattern in which a large area received very nearly the same rainfall in a 24-hr period; and the August storm illustrates a spotty pattern with large ranges in amount in small distances.

A wide spacing of stations would suffice to give a fairly accurate value of the average frequency of rainfall quantities if all storms were of the large-area, homogeneous type shown in Fig. 1(*a*); but a more dense net would be necessary to obtain the same degree of accuracy for the quantities associated with a spotted pattern like Fig. 1(*b*). Since the storms of more uniform intensities, in general, are those of low intensities, the frequencies will be more accurately given for the smaller amounts. Very high rainfall may occur over large areas and give a fairly uniform pattern. Such storms are rare, however, and produce the widespread major floods such as that experienced in the Ohio Valley in January, 1937. A few stations would suffice to give a good sample of the intensities in such storms, but very many years of record are required to give reliable values of their frequency. On the other hand, there are many occurrences of high intensities equivalent to those of the widespread storms, but associated with the spotty type of rainfall, in which there may be several centers of small area receiving very heavy rainfall. The many flash floods that occur throughout the United States every year, washing out soil conserving terraces and small dams, result from this spotty type of rainfall. Although more frequent than the storms producing the major floods, their area is so small relative to the average spacing of stations that many occurrences are not observed and consequently their frequencies cannot be determined accurately.

When several years of record are available from a battery of closely spaced stations, such as that operated in Ohio by the Soil Conservation Service, it will be possible to make comparisons of the average frequencies derived from various spacings and locations of stations, and consequently to provide corrections to be applied to the frequencies obtained from widely spaced, poorly distributed sets of recording stations, in regions with rainfall characteristics similar to the Ohio area.

The Length of Independent Record.—The second factor to be considered in estimating the reliability of an average frequency of specified rainfall is the length of record from which the average was determined. It is obvious that the reliability of a mean frequency based on data from a single station is a function of the length of record for that station, because the longer the record the more occurrences there will be upon which to base a frequency calculation. This must be true, also, for a mean frequency based on the data from several stations in an area. The station-year method simulates a long record for a single station by placing contempory records, from several stations, end to end.

Although by this method more storm experiences may be obtained than in a record for a single station, it obviously cannot be assumed that more actual years of time are being sampled by this expedient. A record from 10,000 rain gages operating on an area for one year cannot replace 10,000 actual years of record for one station although both records could conceivably include the same number of storm experiences. The reliability of an average frequency computed from these two 10,000-station-year records might be the same, but only if each rainstorm of given intensity affected only one station.

The use of additional station records does not increase the reliability of the frequency determinations in direct proportion to the number of station records that are used. An average frequency for an area calculated from a station-year record for several stations, however, is more reliable than one calculated from the record of one of them. Because of the spottiness of rainfall, certain precipitation amounts will be recorded at some stations and not at others during a period of years, and the inclusion of records from many stations will increase the likelihood of including in the calculations of the mean frequency these more rare events.

Regardless of how many station years of record are used, there will always be a certain degree of unreliability in the frequency determinations simply because the actual years of record are not sufficient to sample all the possible annual variations in the rainfall characteristics. If every year had the same characteristics and if there were no abnormally wet or abnormally dry years, then, of course, any one year of record would give as reliable values as any other year. Furthermore, if there are cyclic or secular changes in rainfall, the frequencies from data obtained during a part of a cycle or period may not be applicable to other parts of the cycle. The Miami engineers recognized that the station-year method was valid only if "there are no permanent or cyclic climatic changes affecting the occurrence of high rates of rainfall."[10] This might have been expressed in another way by saying that there will be a high degree of unreliability in the frequency values if there are cyclic or permanent changes in the occurrences of high rates of rainfall.

A quantitative measure of the reliability of average frequencies is needed in order to compare the reliability of the frequency value obtained at a single station with that derived from the composite record of several stations.

In any sampling process the reliability of an average is proportional to the square root of the number of observations upon which the average was determined. The conventional method of expressing the reliability of a mean is in terms of its expected fluctuations, these fluctuations being measured by the standard deviation. The standard deviation of a distribution of means is called the standard error of the mean, and for a normal distribution it is equal to the standard deviation of the sample divided by the square root of the number of values entering the mean. Certain definite relations exist between the size of the standard error and the number of values in the distribution that differ from the mean for the entire population by certain specified amounts. From these established ratios it is possible to indicate the extent to which additional samples may be expected to vary from the average obtained from a single sample. Suppose, for example, that in a 500-yr rainfall record for a

single station, 100 occurrences of a certain amount are observed, giving an average of one occurrence in 5 years, and that the standard error of this mean is 10% of 5 years. Then, knowing the relationships between the size of the standard error of the mean and the distribution of values in the normal curve, one could say that there are 99.7 chances out of 100 that the mean computed for these 500 years (if they are random) is not more than three times the standard error or 30% away from the true average for many thousand years; and there are 68.3 chances out of 100 that this mean will not exceed or fall short of the true mean by more than the standard error, 10%.

The usual formula for the standard error of the mean cannot be applied to frequencies of rainfall because they do not form a normal distribution but rather a discontinuous one of the Poisson type. For this distribution type the standard error expressed as a percentage of the mean is equal to $\frac{1}{\sqrt{n}}$, in which n is the number of independent occurrences.[13]

For the foregoing example, the calculation of the average frequency and its reliability would be as follows: $\frac{500}{100} \pm \frac{1}{\sqrt{100}}$; or 5 years ± (10% of 5); or 5 years ± 0.5 year.

The maximum rainfall that has occurred in any record will have an average frequency (if an average can be said to be determined from a single observation) of once in the total years of record. When the number of observations, N_o, upon which the mean is based is small, generally less than thirty, the usual $\frac{1}{\sqrt{N_o}}$-law fails to give a reasonable estimate of the error, and $\frac{1}{\sqrt{N_o - 1}}$ provides a better though not rigorous approximation (it is inappropriate to discuss herein the difficulties involved in determining the errors in means derived from a few observations. Detailed consideration of the problem has been given by W. N. Bond[14] in 1935 and by G. Udny Yule[15] in 1936). The error of the frequency of the maximum rainfall amount, therefore, would be given by $\pm \frac{1}{\sqrt{1 - 1}}$ or ∞. This can be interpreted only as meaning that a single observation gives no clue to the size of its error; in other words, nothing actually is known concerning the frequency of a rainfall quantity from a single observation of its occurrence. Very high degrees of unreliability are encountered in the expectancies of all high rainfall quantities, inasmuch as the greater the depth the smaller will be the number of occurrences.

Determining the reliability of an average frequency for a single station record is a relatively simple statistical procedure. The next problem concerns its determination for a composite station-year record.

Referring again to Fig. 1(*b*), one sees that during this storm period 5 stations received 3.5 in. or more of rain. If each of the 500 rain gages in this area continued operation for one year, there would be 500 station years of record. Now, assume that 100 storms like that of August 9–10 occurred in

[13] "Statistical Methods for Research Workers," by R. A. Fisher, London, 6th Ed., 1936, pp. 56–59.

[14] "Probability and Random Errors," by W. N. Bond, London, 1935, pp. 45–65.

[15] "An Introduction to the Theory of Statistics," by G. Udny Yule, London, 1936, pp. 277–278, and 353.

this watershed during one year. There would be 500 occurrences (100 storms each affecting 5 stations) in 500 station years, giving an average frequency of one occurrence of 3.5 in. per year. If in this one year of record, instead of 100 storms, there were 500 separate storms each affecting only one station, or if there were one large storm covering the entire area and giving 3.5 in. at all 500 stations, the total number of occurrences of 3.5 in. would be the same, and likewise the average frequency of such an amount would be the same. The station-year method will give the same average frequency for a given number of station years and total occurrences regardless of the combination of numbers of storms and numbers of stations receiving the given amount which makes up the total occurrences.

In so far as the average frequency will indicate how often, on the average, one may expect storms of 3.5 in. in a 24-hr period, however, it would probably be agreed that more reliance could be placed in an average frequency determined from 500 storms than in one based on only one storm. A single observation of a storm in a period of record, regardless of how many stations recorded this storm, tells nothing about its frequency. The storm that resulted in the great Ohio flood of March, 1913, provided 11 of the 16 six-day amounts of 8.7 in. or more of rain in a 1,650 station-year record.[16] The average frequency of this 8.7-in. rain is given as once every 100 years; but the fact that this storm occurred once in 1,650 station years (or even 16,500 station years, if there had been 10 times as many gages in the area) does not indicate whether it can be expected once every 100 years, or once every 1,000 years. In other words, the reliability of an average frequency, expressed as a standard error, should not be calculated from the total occurrences of a given amount in a station-year record, but rather from the number of storms.

In studies involving the station-year method, no reference has been found concerning the reliability of the pluvial indexes; in fact, the number of storms of given intensity is not recorded, and hence standard errors for the average frequencies, based on the number of storms, cannot be determined.

The rainstorm itself is really the independent event whose frequency one would like to know, but since the available data are not in such a form that storm frequencies can be calculated, it is assumed in this paper that the 24-hr rainfall period constitutes the storm, and is the independent event.

If it were true that all storms that result in a given rainfall at any point in an area were of the same areal extent, and if there were equal and fixed distances between recording stations so that the same number of stations recorded the specified amount in every such storm that affected the area, then one could simply divide the total occurrences of that amount in the station-year record by that number of stations and find the number of storms—that is, the number of independent events upon which the reliability of the average frequency should be based. Since storms of a given intensity do not always affect the same number of stations, the average number of stations affected by storms of this intensity may be used instead to determine the number of storms. This average number of stations will be designated simply as N_a herein.

[16] "Storm Rainfall of Eastern United States," Miami Conservancy District, Eng. Staff, *Technical Reports*, Pt. 5, Revised Edition, 1936, Dayton, Ohio, p. 66.

Consider again the case of 100 occurrences of rainfall of certain depth, this time occurring in a station-year record of 500 years derived from 20 years of record for 25 stations. Suppose, in this case, that the average number of stations recording the given amount in each rain period of 24 hours were five. There were, then, $\frac{100}{5}$ or 20 independent rain periods. The average frequency and its standard error would be given by $\dfrac{25 \times 20}{100 \pm \dfrac{1}{\sqrt{20}}}$; or 5 years $\pm$ 22%; or 5 years $\pm$ 1.1 years.

Compared with the average frequency for the 500-yr record for a single station, which had a standard error of $\pm$ 0.5 year, one notes that, although the frequency remains the same, the reliability is less when determined from the number of storms in a station-year record than when determined from the single station record of equivalent length.

It has been shown that the reliability of the average frequency should be determined from the number of independent events (that is, from the number of storms), and that this number may be obtained by dividing the total number of occurrences in the station-year record by N_a. The fact that frequently more than one station records the same storm may be described as dependence between stations. One phase of this problem of determining the reliability of the average frequency of rainfall quantities becomes, then, one of calculating the dependence between recording stations.

Determination of Dependence Between Stations

Although it is possible to use the average number of stations per storm receiving the given rain quantity as the index of dependence between stations, another measure of dependence will be described herein that is an even better index upon which to base the reliability determinations. This index is calculated by a statistical technique, devised by J. Bartels,[17,18] and proposed originally for testing persistence in observations taken at equal time intervals. (A series of numbers having "persistence" is called a correlative series. There is a tendency in such a series for high values to be followed by high values, and low values to be followed by low values.) In the case of a time sequence successive observations are affected by a single event because of its extension in time; in the station-year record of rainfall amounts, adjacent stations may be affected by a single event because of the areal extent of the storm.

The technique for testing persistence is based on the fundamental theorem of probability that in a random series of numbers the ratio of the standard deviation of the means of equal-sized groups of these numbers to the standard deviation of the individual numbers of the series is equal to unity divided by the square root of the number of values in the groups ("randomness," as used herein, means that, on the basis of the numbers up to a certain point, the chance for the next number to be higher or lower than the mean for a long series is equal or unpredictable. A series of random numbers can be most

[17] "Zur Morphologie Geophysikalischer Zeitfunktionen," by J. Bartels, Sonderausg. aus den *Sitzber.* der Preussischen Akad. der Wiss. Phys.-Math., Klasse 30, 1935.

[18] "Geomagnetism," by S. Chapman and J. Bartels, Oxford, Vol. II, 1940, pp. 582–585.

easily obtained from a table of "Random Sampling Numbers," by L. H. C. Tippett[19]).

This may be explained more clearly by carrying through the calculations on a series of random numbers. One first marks off groups of a certain number of ordinates and forms the average for each of these groups. These group averages will approximate the normal more closely than the individual values. If, for example, groups of 25 ordinates were used, the scattering of the group averages measured by their standard deviation, σ_{25}, will be less than that indicated by the standard deviation, σ_1, for the individual numbers; but $\sigma_{25} : \sigma_1 = 1 : \sqrt{25}$; or $\sigma_{25} = \frac{\sigma_1}{\sqrt{25}}$. In general, if groups of s numbers, instead of 25, are formed, $\sigma_s = \frac{\sigma_1}{\sqrt{s}}$.

Returning to the theoretical case of 25 stations, each having 20 years of record, and storms of given depth always affecting 5 stations in each storm: From the 20 years of record for these 25 stations one may express the relationship between the standard deviations of the total occurrences for each year in the groups of 25 stations, and the standard deviations of the number of occurrences at the individual stations each year as

$$\sigma_{25} = \frac{\sigma_1}{\sqrt{\frac{25}{5}}} \qquad (1)$$

Substituting s for 25 and N_d for 5 in Eq. 1:

$$N_d = \left[\frac{\sigma_s}{\frac{\sigma_1}{\sqrt{s}}}\right]^2 \qquad (2)$$

in which N_d equals the number of stations on the average receiving the given amount in each storm day.

To show how this technique may be applied to a set of rainfall data, assume that one had 10 years of concurrent record for 10 stations. The total occurrences of a certain rainfall amount at each station for each year could be set up in a table of 100 values, with the 10 columns representing stations and the 10 rows representing years. If there is no dependence between stations or years, these 100 values will constitute a random series. Any departure from randomness will be a measure of the amount of dependence between the stations, or between years.

Artificial tables of rainfall data have been set up by using random numbers and introducing a known degree of dependence between stations.[20] Eq. 2 was tested by calculating N_d from such tables. In every case the correct value of the dependence within the standard error of N_d was obtained (the standard error of N_d was calculated on the assumption of a normal distribution

[19] "Random Sampling Numbers," by L. H. C. Tippett (Tracts for Computers 15, London Univ., Dept. of Applied Statistics, 1927).

[20] "On the Bartels' Technique for Time-Series Analysis, and Its Relation to the Analysis of Variance," by L. R. Hafstad, *Journal*, Am. Statistical Association, Vol. 35, 1940, pp. 347–361.

since any error due to this assumption will be of second order. A more detailed consideration of the calculation of the standard error of a non-normal distribution is out of place in a non-mathematical paper such as this).

The χ^2-test and analysis of variance were tried on the table of random numbers, but since they gave only the information that the numbers in the rows were more alike than one would expect in a random sample, and that the numbers in the columns were about as in a random sample, these tests were discarded as inadequate for the determination of N_d, the number of dependent ordinates, or stations.[20]

If the average number of stations recording the given depth in a storm day were the same for every year, and if the numbers of storm days per year constituted a random series, then the value of N_d would be equal to N_a for the total years of data.

In actual rainfall data, a variability exists from year to year in the average number of stations affected per storm, and a greater fluctuation exists in the number of storm days than might be expected in a random series. The value of N_d is determined from the sums of the occurrences for each year, and there are wide fluctuations in these annual sums because they combine the fluctuations in the average for each year of the number of stations affected per storm, and the annual fluctuations in the number of storms. If there are wide fluctuations from year to year in the number of storms, then it is necessary to have more years of data to give an average frequency that is as reliable as would be obtained from a few years which had practically the same number of storms each year.

In addition to measuring the persistence between stations (average number of stations affected per storm), N_d measures also these fluctuations from year to year, and it is therefore a more conservative, but also a more accurate, index to use in calculating the reliability than simply the average number of stations affected per storm. The standard errors will be higher if determined from N_d than if determined from either the total occurrences or the number of storms.

USE OF N_d FOR ACTUAL RAINFALL DATA IN DETERMINING STANDARD ERRORS OF FREQUENCIES OF RAINFALL AMOUNTS

The calculation of N_d has been made from rainfall data for two areas in the United States in different climatic regions. The method of tabulation and calculation will be described in detail for one of these areas.

Twenty-one stations in the quadrangle between meridians 93° and 95° west longitude and parallels 41° and 43° north latitude in central Iowa, corresponding to Quadrangle 14-D of the Miami study,[4] were selected to represent that region. Tabulations were made of all daily rainfall depths of one inch or more at every station for 32 years, 1905 through 1936. Only stations at which observations were made in the evening, and only those stations having continuous records for the entire period, were used.

Tables were made of the total number of occurrences at each station during each year of daily rains of one inch or more, 2 in. or more, 3 in. or more, and 4 in. or more. Table 1 is a sample tabulation and compilation

TABLE 1.—THE NUMBERS OF OCCURRENCES OF DAILY RAINS OF ONE INCH OR MORE AT 21 STATIONS IN CENTRAL IOWA FOR THE YEARS 1905–1936, AND DATA FOR CALCULATING N_d

Year	Ames	Afton	Baxter	Belmond and Britt	Chariton	Cumberland	Monroe and Des Moines	Newton and Grinnell	Guthrie Center	Hampton and Allison	Humboldt	Jefferson	Knoxville	Lacona	Perry	Pocahontas	Rockwell and Sac City	Waukee	Webster	West Bend	Winterset	Σ	$\frac{\Sigma}{N}$	$\left(\frac{\Sigma}{N}\right)^2$
1905	9	11	9	5	15	15	10	10	7	10	9	5	12	8	9	16	8	15	9	8	10	210	10.000	100.000
1906	9	4	3	8	5	7	2	5	5	7	11	2	8	3	7	8	12	5	8	11	8	138	6.571	43.178
1907	4	7	10	6	9	10	4	10	4	9	7	7	6	9	8	7	5	13	6	5	6	152	7.238	52.389
1908	12	14	6	11	6	17	9	6	7	8	12	5	8	10	10	10	20	13	7	10	11	212	10.095	101.909
1909	14	18	8	10	10	9	7	10	9	12	12	7	10	6	19	10	16	10	11	11	5	224	10.667	113.785
1910	5	7	5	3	1	9	5	2	7	3	4	1	5	2	3	1	8	2	4	2	4	83	3.952	15.618
1911	9	3	5	5	9	4	6	11	4	10	5	2	6	5	4	6	6	5	9	5	8	127	6.048	36.578
1912	8	10	8	6	6	6	6	9	13	9	1	6	6	9	6	9	11	6	3	7	7	152	7.238	52.389
1913	5	10	11	5	6	10	1	11	3	11	6	6	5	7	4	5	14	6	6	10	8	150	7.143	51.022
1914	9	5	11	12	5	5	10	10	9	9	5	11	7	6	9	9	7	8	6	8	9	170	8.095	65.529
1915	11	11	8	8	14	11	12	13	9	8	10	5	13	12	10	8	8	10	9	9	8	207	9.857	97.160
1916	4	4	7	10	7	2	6	6	8	7	3	8	5	6	5	7	4	1	5	6	5	116	5.524	30.515
1917	12	6	9	6	11	6	10	10	9	9	6	9	9	6	9	5	5	6	8	9	8	168	8.000	64.000
1918	6	6	10	11	5	5	11	11	5	8	3	7	3	5	4	6	6	6	9	8	6	141	6.714	45.078
1919	9	13	8	9	12	10	13	12	12	9	5	9	12	12	13	3	8	14	10	4	12	209	9.952	99.042
1920	6	10	5	9	9	7	15	4	8	9	11	13	9	9	7	5	7	9	3	9	5	169	8.048	64.770
1921	7	6	4	4	12	5	12	13	7	12	6	10	8	10	7	7	5	6	2	7	8	158	7.524	56.610
1922	6	9	11	4	9	8	10	5	5	8	8	8	12	12	4	5	8	8	8	5	13	166	7.905	62.489
1923	5	3	9	4	3	6	9	4	9	6	5	8	8	2	3	5	5	4	3	6	3	110	5.238	27.437
1924	6	10	8	8	4	11	6	8	9	6	7	5	6	9	2	6	5	9	8	5	9	147	7.000	49.000
1925	8	8	6	7	9	3	6	7	5	8	3	3	8	9	7	2	4	8	6	2	3	122	5.810	33.756
1926	7	9	6	8	10	5	10	6	8	3	6	8	10	10	8	4	6	6	4	7	5	146	6.952	48.330
1927	3	4	5	4	6	3	4	4	3	4	5	4	4	9	4	1	2	5	3	3	2	82	3.905	15.249
1928	10	11	7	11	8	9	13	10	10	11	9	9	8	14	12	7	6	8	5	5	8	191	9.095	82.719
1929	7	6	5	4	4	7	6	2	3	8	6	2	1	5	5	4	4	8	8	5	3	103	4.905	24.059
1930	3	8	4	4	5	2	6	5	2	7	4	3	6	5	2	4	8	1	6	4	8	97	4.619	21.335
1931	9	11	10	7	16	9	10	8	9	8	4	10	10	15	7	7	13	16	5	7	12	203	9.667	93.451
1932	8	9	9	8	5	13	9	10	8	6	9	6	6	14	6	10	8	6	8	8	5	171	8.143	66.308
1933	3	7	6	7	6	3	4	3	4	4	8	6	4	7	5	8	7	3	7	8	3	113	5.381	28.955
1934	7	5	6	6	4	3	5	8	4	8	5	6	7	4	4	7	6	6	7	4	3	115	5.476	29.987
1935	10	11	8	6	14	7	14	12	6	7	8	7	16	15	7	4	4	9	7	8	14	194	9.238	85.341
1936	6	5	8	9	7	8	9	7	7	6	6	5	5	8	7	3	2	8	4	5	8	133	6.333	40.107
Totals (Sum of the squares of the individual values = 42,041)																						4,879	232.333	1,798.095

sheet, this one being for rains of one inch or more in 24 hours. By substituting the values from Table 1 in the formulas, the values of N_d were computed as follows:

$$\sigma_{21} = \sqrt{\frac{1{,}798.095}{32} - \left(\frac{232.333}{32}\right)^2} = \sqrt{3.4771}$$

$$\sigma_1 = \sqrt{\frac{42{,}041}{672} - \left(\frac{4{,}879}{672}\right)^2} = \sqrt{9.8476}$$

$$N_d = \left(\frac{\sqrt{3.4771} \times \sqrt{21}}{\sqrt{9.8476}}\right)^2 = 7.415.$$

The corresponding standard error equals 1.854. Calculations of N_d were made for rains of various intensities only up to 4 in. or more per day. Heavier rains were so few that N_d could not be determined with any degree of reliability.

Similar tabulations and calculations were made from 20 years (1917 through 1936) of rainfall data for 15 stations in the quadrangle between parallels 33° and 35° north, and meridians 79° and 81° west, comprising parts of North and South Carolina. This area is Quadrangle 7-H of the Miami study.

Table 2 gives the values of N_d and of N_a derived from the data for the areas

TABLE 2.—VALUES[a] OF N_d AND N_a, WITH THEIR STANDARD ERRORS, FOR VARIOUS RAINFALL DEPTHS, IN A 2° QUADRANGLE IN IOWA AND IN NORTH AND SOUTH CAROLINA

Rainfall, in inches per day	Iowa (21 Stations, 32 Years)		North and South Carolina (15 Stations, 20 Years)	
	N_d	N_a	N_d	N_a
One or more	7.42±1.85	3.52±0.58	8.49±2.68	3.10±0.57
Two or more	4.20±1.49	2.05±0.47	6.04±1.90	1.95±0.47
Three or more	1.46±0.73	1.72±0.40	3.90±2.08	1.72±0.41
Four or more	1.00 (approximately)	1.37±0.31	1.70±1.07	1.68±0.78

[a] N_d equals number of stations to be used in calculating the reliability of rainfall frequencies, and N_a equals average number of stations affected by storms of a given intensity.

in Iowa and in South Carolina. The values in the table show that, in general, the more intense storms cover a smaller area than those yielding lower quantities. When N_d or N_a approximate unity, the occurrences may be considered to be random events. On the average an occurrence of 3 in. of rainfall will affect only one station per storm day in Iowa, but in South Carolina about four stations would be affected, and this in spite of the fact that the stations are more widely spaced in South Carolina.

There are three bases upon which one might compute a standard error of the average frequency of specified amounts of rainfall. One of these is the sum of the occurrences, which is the one assumed by the station-year method. Another is the number of storms, which is equivalent to the total occurrences divided by the average number of stations affected. The third uses N_d in

place of average number of stations. The differences in standard error obtained by the first and the second of these methods have been shown earlier in the paper for a theoretical case. In Table 3 these comparisons are made for the

TABLE 3.—COMPARISON OF THE STANDARD ERRORS OF RAINFALL FREQUENCIES BASED ON TOTAL OCCURRENCES, ON NUMBER OF STORM DAYS, AND ON N_d

Rainfall, in inches per day	(a) IOWA				(b) SOUTH CAROLINA			
	Frequency (years)	Standard Error (%) by:			Frequency (years)	Standard Error (%) by:		
		Station-year method	Storm days	N_d		Station-year method	Storm days	N_d
	(1)	(2)	(3)	(4)	(1)	(2)	(3)	(4)
One or more...	0.14	1.4	2.7	3.9	0.08	1.6	2.8	4.7
Two or more...	0.80	3.4	4.9	7.1	0.38	3.6	5.1	8.8
Three or more..	3.10	6.8	8.8	8.2	1.47	7.0	9.2	13.9
Four or more..	10.67	12.6	14.7	14.0 (approximately)	3.80	11.2	14.1	14.7

actual rainfall data for the areas in Iowa and the Carolinas, the computations being:

Column Nos.	Computation
1	$\dfrac{\text{Stations} \times \text{years}}{\text{Occurrences}}$
2	$\pm \dfrac{1}{\sqrt{\text{Occurrences}}}$
3	$\pm \dfrac{1}{\sqrt{\left(\dfrac{\text{Occurrences}}{\text{Average number of stations}}\right)}}$
4	$\pm \dfrac{1}{\sqrt{\left(\dfrac{\text{Occurrences}}{N_d}\right)}}$

The availability of either the value of N_a or N_d makes possible an approximate answer to the question proposed by C. S. Jarvis,[21] M. Am. Soc. C. E., and others regarding the equivalence of a station-year record to a record of the same length for a single station. In the Carolina quadrangle, for rainfalls of one inch or more per day, on the average 3 stations record that amount in a rain day. As a result there are only $\frac{15}{3}$ or 5 independent stations. The station-year record for this quadrangle of 300 station years (15 stations for 20 years each) could not be considered equivalent to 300 years for a single

[21] *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 394.

station, but would be more nearly equivalent to 5 × 20 or 100 years; one might call this the valid station years of record based on the average number of stations affected per storm. For the foregoing reasons it would be still better to use the value N_d rather than N_a. This would give 36 valid station years for rainfalls of one inch for the Carolina quadrangle.

Standard Errors for Frequencies of Pluvial Indexes of the Miami Study

Although the true reliability of frequency values should be determined on the basis of the degree of persistence in the data, it is possible to give some indication of the reliability from the total number of occurrences of the given rainfall depth appearing in the station-year record.

A practical application of the standard error as a measure of the reliability of rainfall frequencies may be made in connection with the charts of pluvial indexes for quadrangles in Eastern United States, published by the Miami Conservancy District.[11] For each 2° quadrangle these charts give the maximum rainfall (one day to six days) to be expected once in 15 years, 25 years, 50 years, and 100 years. There is given, also, for each quadrangle the total number of station years used in the computations.[22] The number of station years divided by 100 gives the rank of the 100-yr pluvial index in the series of rains arranged in order of depth. For example, for Quadrangle 8-E (see Fig. 2(*d*)) in western Ohio, the pluvial-index of 5.2 in. for the 100-yr frequency has a rank of 15, obtained by dividing 1,524 (station years) by 100. In other words, this pluvial index with an expectancy of once in 100 years was based on 15 occurrences. A percentage standard error in the frequency may then be given as $\frac{1}{\sqrt{15}}$, or 26%, and the average frequency should be written as 100 years ± 26 years. A percentage standard error on the basis of all occurrences has been calculated for all quadrangles of the Miami study for the 100-yr, 50-yr, 25-yr, and 15-yr frequencies, and these percentages are shown in Fig. 2. Errors were calculated from the total occurrences with no correction for dependence in the data.

Since the number of occurrences will depend upon the intensity, the number of years of record, and the number of stations, the reliability of the frequency will be least for high rainfall amounts in quadrangles with the fewest station years of record. It will be seen that about one fifth of the quadrangles have standard errors of 50% for the 100-yr pluvial index. For these quadrangles the expectancy should be given as once every 100 years ± 50 years.

Summary

It has been shown that two factors affect the accuracy of average frequency values of precipitation determined by the station-year method—the number and distribution of stations and the length of independent record. The second of these factors has been discussed in some detail. It has been shown

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One or more...	0.14	1.4	2.7	3.9	0.08	1.6	2.8	4.7
Two or more...	0.80	3.4	4.9	7.1	0.38	3.6	5.1	8.8
Three or more..	3.10	6.8	8.8	8.2	1.47	7.0	9.2	13.9
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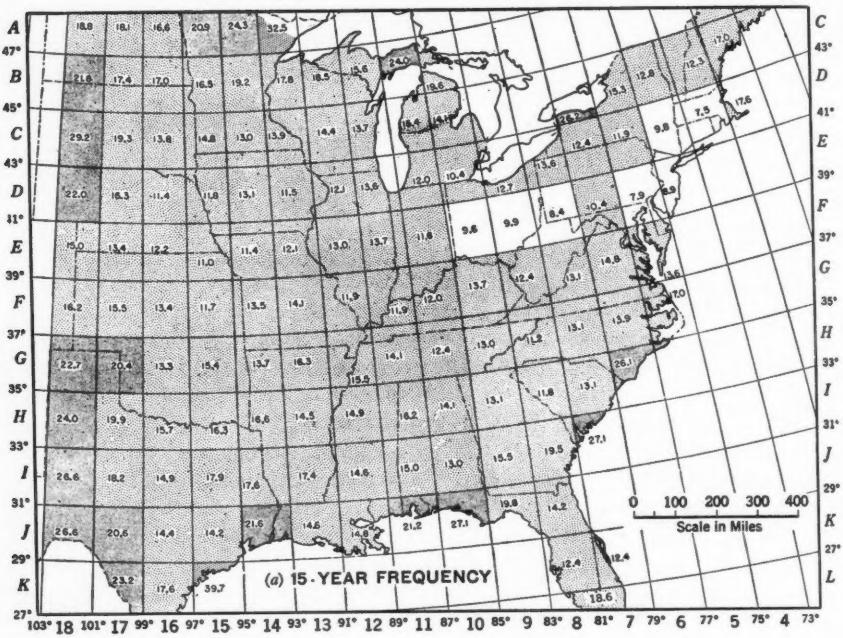


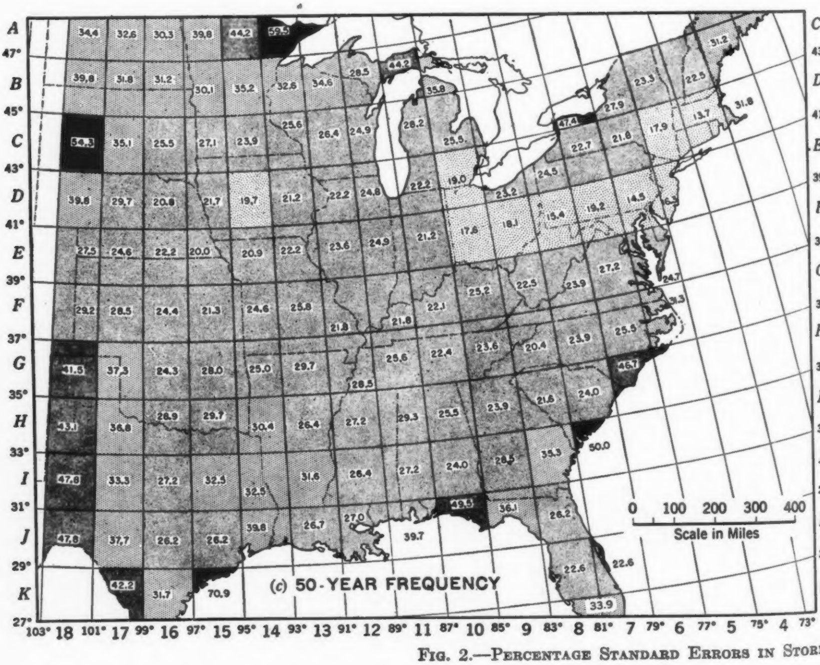
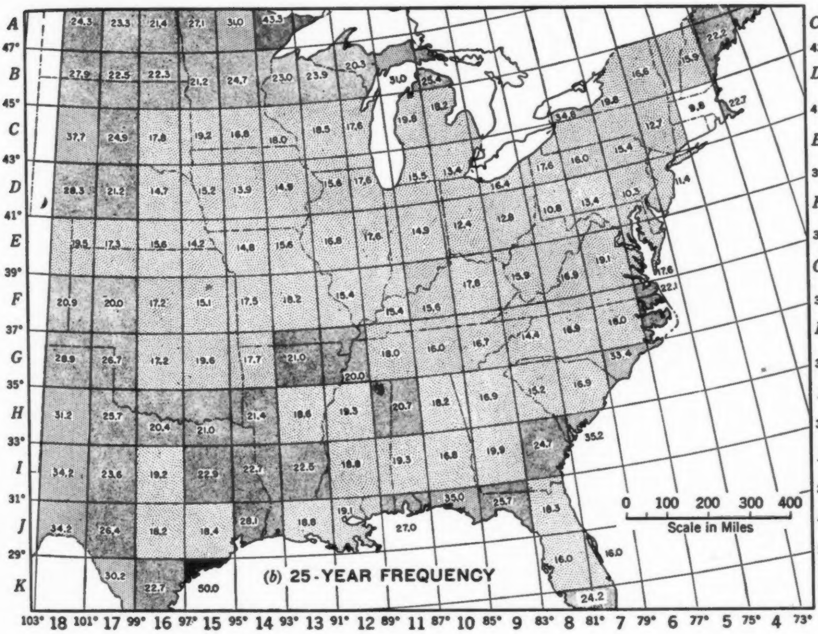
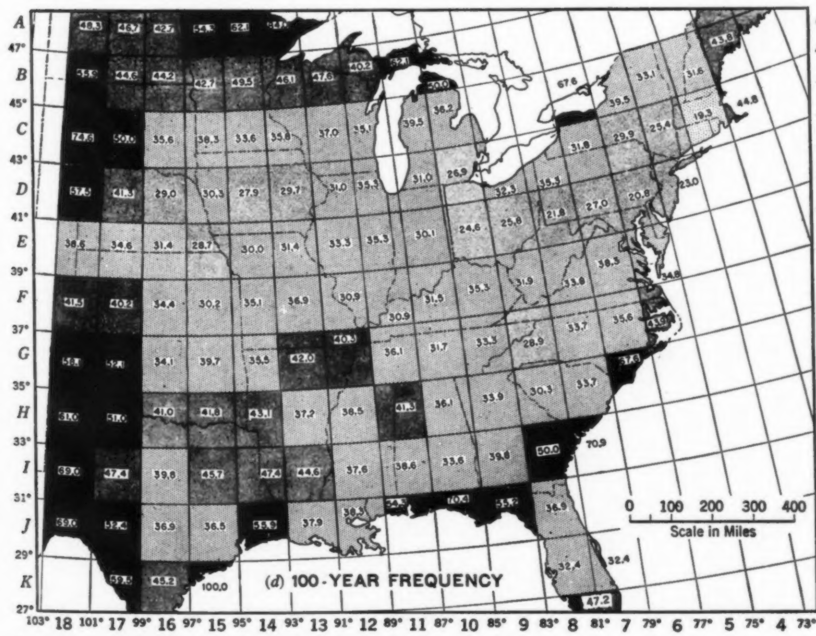


FIG. 2.—PERCENTAGE STANDARD ERRORS IN STORM-





FREQUENCY DETERMINATIONS OF MIAMI STATION-YEAR STUDY

that the reliability of the frequency, as far as it is related to the length of record, can be expressed as a percentage error in the average frequency. An index of dependence, N_d, has been proposed as the best basis yet devised on which to compute the standard errors.

A measure of the reliability of frequency-intensity values can be helpful to those concerned with the design of soil-conservation and flood-control structures, since the margin of safety to be allowed in design should be in proportion to the reliability of the frequency determinations.

Little is known about the effect of station spacing and distribution upon the average frequency, primarily because it has not yet been possible to determine accurately the areal extent of storms of certain intensity. The question of the adequacy of sampling of rainstorms, also, depends upon the size and structure of rainstorms in relation to the spacing of rain gages. The fundamental data for studying rainstorm morphology are only now becoming available through the establishment of dense networks of rain gages such as those in the Susquehanna, Muskingum, and Tennessee watersheds.

New projects in the study of rainfall frequencies and intensities, therefore, will make use of these additional data in considering the sizes of storms and consequently the adequacy of sampling with various spacings of recording stations. A great weakness of the station-year method is its failure to indicate the areal extent of rainfalls of certain intensity and frequency. Studies of rainstorm morphology should supply this deficiency by making possible the determination of the maximum depths of rainfall which will cover a certain area and occur with a given frequency. The probable frequency of specified volumes of runoff can then be determined more accurately than can be done now with a pluvial index referring only to a point.

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PAPERS

CAVITATION IN OUTLET CONDUITS OF HIGH DAMS

By Harold A. Thomas,[1] M. Am. Soc. C. E., and Emil P. Schuleen,[2] Assoc. M. Am. Soc. C. E.

Synopsis

Occurrence of severe cavitation damage to the concrete surfaces of the outlet conduits of the Madden Dam, in the Panama Canal Zone, supplied the incentive to carry on extensive studies by means of laboratory models to investigate the cavitation potentialities, if any, in the conduits of the Tygart River Dam, near Grafton, W. Va., and to develop methods of eliminating or minimizing future damage in the conduits of the Madden Dam. Similar studies, on a less elaborate scale, were conducted in connection with the design of the Bluestone Dam in West Virginia, Hiwassee Dam in North Carolina, and Redbank Creek Dam in Pennsylvania.

The development of cavitation-testing facilities of two types, known respectively as the "enclosed-tank apparatus" and the "diverging-tube apparatus," is described, and the hydraulic theory pertinent to the making of cavitation tests in these facilities is presented. A description is given of cavitation studies conducted on models of the conduit entrances of the Madden Dam.

Introduction

In the late summer of 1935 the pouring of concrete in the lower portions of Tygart River Dam was in progress under the direction of the district engineer, U. S. Engineer Office, Pittsburgh, Pa., when word was received from the Panama Canal authorities that the concrete in the entrance portions of the conduits in Madden Dam had been seriously damaged by cavitation. Since the conduits to be placed in Tygart River Dam were identical in cross section to those in Madden Dam, and differed only slightly in entrance curvature, concern was naturally felt regarding the possibility of cavitation also occurring

Note.—Written comments are invited for immediate publication; to insure publication the last discussion should be submitted by **March 15, 1941.**

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in the Tygart River Dam conduits. To test the design for such a possibility, and to provide means for analyzing possible alterations in design to eliminate cavitation, should such be found to exist, arrangements were made with the Hydraulic Laboratory of the Carnegie Institute of Technology, in Pittsburgh, to construct and test a model of the Tygart conduit entrance.

The tests to be undertaken under this arrangement were of an unusual nature for which there was little precedent, since a search of records of cavitation tests disclosed none of models of existing or proposed conduits designed for high velocities. It was necessary, therefore, to design a complete testing apparatus in order that correct scale reproductions of the absolute pressures operating in the prototype could be developed in the model. The pioneering nature of these tests led to some uncertainty regarding the ability to secure an accurate reproduction in the model of cavitation effects which might appear in the prototype if the entrance were improperly designed. As a means of verifying the accuracy with which prototype phenomena were reproduced, a model of the conduit entrance in Madden Dam was constructed and tested, and the results compared with data on cavitation and damage in the prototype as reported by the Panama Canal authorities.

As the tests progressed, the Governor of the Panama Canal Zone took an increasing interest in the work and in the early spring of 1936 officially requested that tests be conducted to develop a feasible means of eliminating or alleviating the cavitation action and resulting damage in the conduits of Madden Dam. By this time, tests of the conduits designed for Tygart River Dam had been substantially completed, so that the testing apparatus could then be made available almost exclusively for tests of models of the conduits in Madden Dam. Various designs of entrance curves for conduits were tested in developing a design that might be readily installed in the prototype at a minimum cost. A suitable design was evolved and a report was in the final stages of preparation when the Governor of the Panama Canal Zone requested a series of additional tests. These additional tests involved: (*a*) A check of the previously developed design to determine its adequacy at somewhat higher heads as well as its adequacy when the two conduits of a pair were operated together (previous tests had disregarded the effect of an adjacent conduit in operation); (*b*) further study of a bellmouth entrance designed by the engineers at the Panama Canal; (*c*) studies of the degree of gate opening that could be permitted without destructive cavitation occurring in the conduit entrance; (*d*) various schemes for increasing the pressure in the critical area by constrictions within the conduit or at its discharge end; and (*e*) various studies involving fillers for stop-log slots, venting of the conduits, etc. The second series of tests was completed during the winter of 1937–1938 and a report was made the following summer.

During and subsequent to the tests at Madden Dam, the availability of the cavitation apparatus at the Hydraulic Laboratory of the Carnegie Institute of Technology permitted the testing of conduit entrances for several other dams, among which may be mentioned the Bluestone, Hiwassee, and Redbank Creek dams.

A description of the details of the numerous tests on the Madden models, and of cavitation studies on models of the outlet conduits of the Tygart, Bluestone, Redbank Creek, and Hiwassee dams, is included in the original manuscript from which this paper is condensed, the original being on file in Engineering Societies Library.[3]

Cavitation Effect in Madden Dam Conduits as Originally Designed and Constructed

The Madden Dam,[4] a concrete gravity structure comprising part of the Panama Canal Project, is pierced by six rectangular conduits, located as shown subsequently in Fig. 12. These conduits are grouped in pairs and are controlled by service and emergency slide gates. Each is 5 ft 8 in. wide and operates under a maximum head of 155.3 ft. The upstream or intake ends of the conduits, which are the points of particular interest in this paper, were flared in the original construction on a radius of 4 ft as indicated by Fig. 1

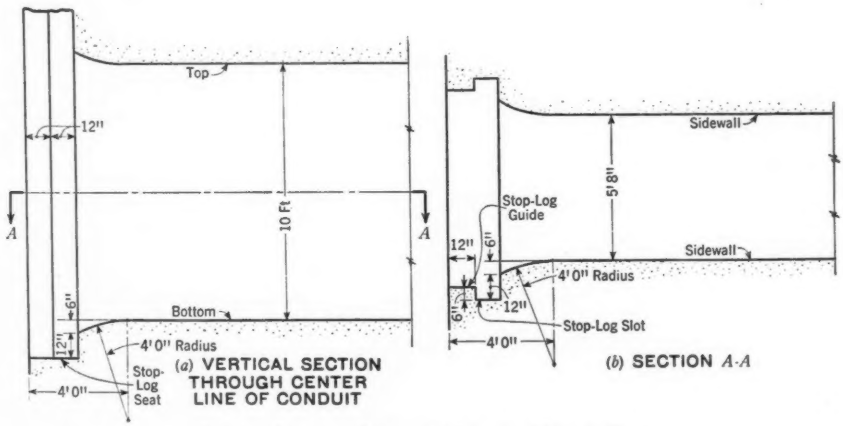


Fig. 1.—Original Conduit Entrance, Madden Dam

(which also shows the design and location of the stop-log slots). The conduits are steel-lined for a reach of 48 ft in the vicinity of the slide gates but are unlined above and below this reach. The intake ends of the conduits, therefore, are unprotected by metal lining over the initial 12.89 ft, this measurement including the stop-log slot and guide.

The reservoir was placed in operation in 1934, and the conduits were in use at various times during the process of filling the reservoir. By March 12, 1935, the water level had been lowered to an elevation one foot above the spillway crest by discharging over the crest while making spillway tests, and the conduits were then placed in operation to continue the release of impounded water. It was observed at once that the operation of the conduits was accom-

[3] 29 West 39th Street, New York, N. Y.

[4] "Hydraulic Tests on the Spillway of the Madden Dam," by Richard R. Randolph, Jr., *Transactions*, Am. Soc. C. E., Vol. 103 (1938), p. 1080.

panied by a loud crackling and popping noise which was particularly noticeable in the slide-gate galleries. However, operation was continued at the full capacity of the conduits until March 15, 1935, when a porous tile drain in the partition wall between Conduits 1 and 2 began to leak water into the gallery. It was then concluded that the crackling noise and the leak which had developed were the result, respectively, of cavitation and its damaging effect in the entrance portion of the conduit. It was decided to reduce the cavitation action by throttling the discharge by partial closure of the slide gates. The crackling noise or crepitation ceased when the gates were closed to the 90% open position, and the gates were left in that position during the remaining process of draining the reservoir.

After the reservoir had been completely drained, the conduits were inspected and the damage that had resulted from cavitation was recorded. The inspection revealed the fact that the cavitation action had seriously eroded the walls and tops, or ceilings, of the conduits by pitting the concrete to depths of as much as almost 2 ft and in many places by exposing reinforcing bars that had originally been 10 in. from the surface of the concrete. The severity of the damage may be noted in Fig. 2, which shows erosion typical of that which occurred in all of the units except Conduit 3. In the latter conduit, damage was slight because it had been operated at high heads for only a comparatively short total period of time. Fig. 2(*a*) is a view of Conduit 1, facing downstream. It shows pitting of the concrete on the top and upper part of the right side-wall. Note the reinforcing bars exposed on the top of the conduit, and also note the beginning of the steel lining on the right side-wall at the left. Fig. 2(*b*) is a view facing toward the intersection of the top and right side-wall at the extreme upper end of Conduit 2 and shows damage to those two surfaces of the conduit. Note the intersections of the top and side-wall entrance curves with the upstream face of the dam. Also note the stop-log slot at the extreme right.

For the most part, the damage was found to be confined to the tops or ceilings and the upper parts of the side-walls of the conduits as is indicated by Fig. 3, which shows, by contours and cross sections, the damage to the top and side-walls of Conduit 5. This also is typical of the damage to all units except Conduit 3.

A résumé of the number of hours and minutes that each conduit was operated at various heads during the first season of operation, and the resulting damage as indicated by the maximum depth of pitting and the volume of concrete disintegrated and removed, is given in Table 1.

Disregarding Conduit 3, where damage was comparatively slight, analysis of the data in Table 1 indicates that the concrete on the tops of the conduits was pitted to maximum depths of more than 1.0 ft to 1.5 ft, whereas the side-walls were pitted to maximum depths of more than 0.5 ft to 1.75 ft. Neglecting the time in which the conduits were operated at 90% gate opening (when, as previously explained, cavitation was reduced to a minimum), the data for Conduits 1, 2, 5, and 6, as given in the table, indicate a progressive increase in the volume of concrete disintegrated and removed with increased hours of operation at heads greater than 45 ft. However, comparison of the data for

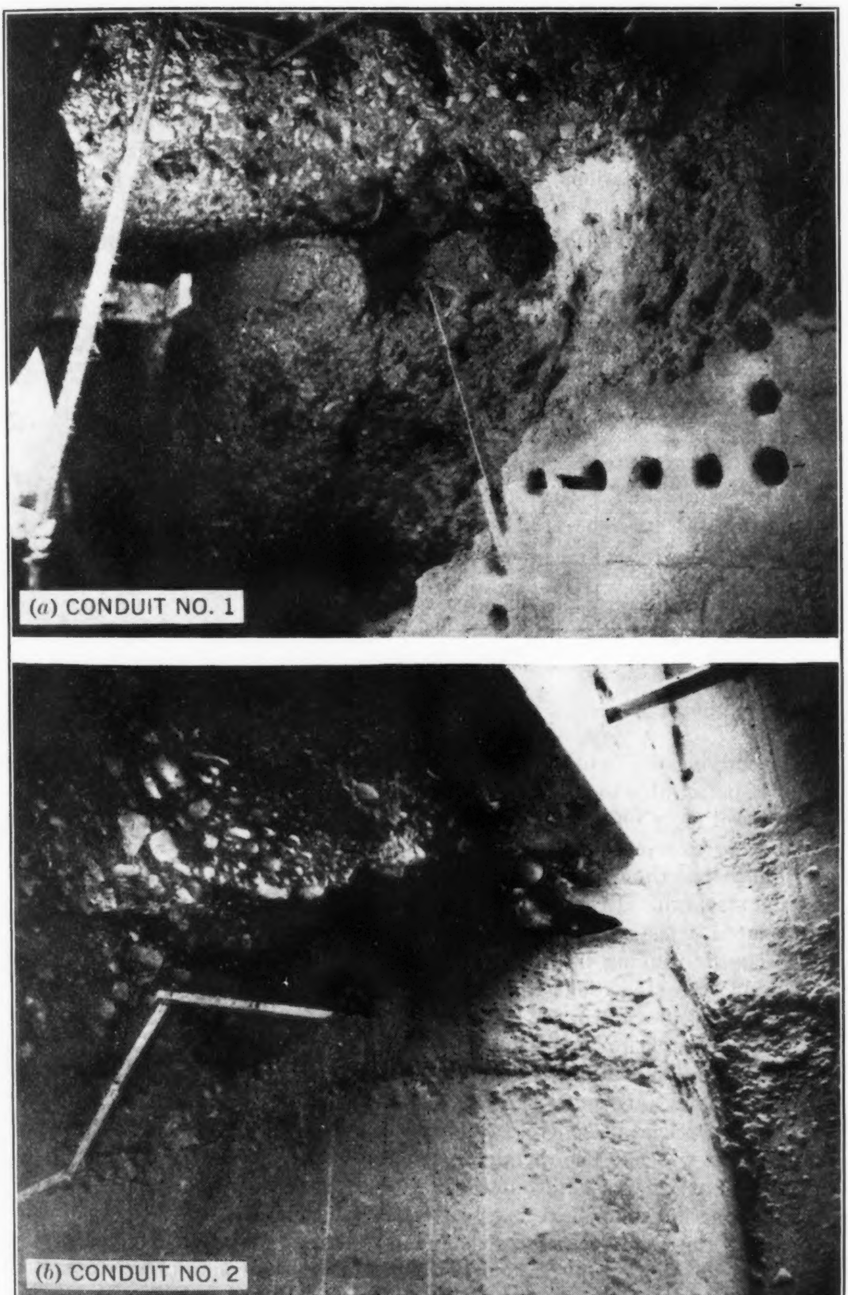


FIG. 2.—VIEWS FACING DOWNSTREAM TOWARD THE INTERSECTION OF THE TOP AND RIGHT SIDE-WALL OF CONDUITS, MADDEN DAM

Conduits 1 and 2 with those for Conduits 5 and 6 shows a tendency for the rate of disintegration and removal of concrete to become less as the conduits are subjected to greatly increased hours of operation at high heads. This may be explained by the tendency for the erosion on the top of the conduit to become stabilized but to continue to progress down the side-walls, as will be discussed more fully in comparing the cavitation region or pocket with the eroded area in the section, "Behavior of Original Madden Entrances in Models."

TABLE 1.—OPERATION DATA AT THE END OF THE FIRST SEASON

Conduit	Hours and Minutes of Operation[a]							Approximate Maximum Depth of Pitting[b] (Ft)			Volume of Concrete Removed (Cu Ft)			
	At Full Gate Opening				At 90% Gate Opening									
	Head Range (Ft)			Total 45–145	Head Range (Ft)		Total 45–145							
	0–45	45–95	95–145		45–95	95–145		Top	Right wall	Left wall	Top	Right wall	Left wall	Total
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)
1	26:03	1:53	136:08	138:01	....	29:08	29:08	1.50	1.25	1.75	27.3	14.2	11.5	53.0
2	84:58	3:41	126:42	130:23	80:01	131:05	211:06	1.25	0.75	1.50	26.4	5.5	9.6	41.5
3	60:06	43:22	18:57	62:19	79:39	119:52	199:31	0.25	0	0	0.4	0.0	0.0	0.4
4	150:58	67:02	165:08	232:10	79:19	131:06	210:25	1.00	1.00	0.50	23.9	4.0	2.3	30.2
5	62:56	231:58	152:59	384:57	78:25	131:02	209:27	1.50	1.50	1.00	33.2	19.8	7.8	60.8
6	196:43	32:35	203:21	235:56	77:08	131:00	208:08	1.25	1.50	1.50	29.3	9.8	16.0	55.1

[a] Includes all operations from the time gates were first closed to impound water until March 28, 1935.
[b] Values given correspond to maximum depth contours shown on drawings for each conduit similar to those of Fig. 3.

In studying the data of Table 1, it may be noted that Conduit 4 was apparently better able to withstand the ravages of cavitation than were Conduits 1, 2, 5, and 6. Conduit 3 was in operation under high heads for only a few hours. No explanation can be given for this inconsistency other than the possibility that the concrete in the region of the entrance to Conduit 4 was of unusual strength. Records of compression tests made on samples of the concrete placed in the regions of the various conduit entrances gave 28-day breaking strengths ranging from about 3,230 lb per sq in. to about 4,680 lb per sq in., and averaging about 3,800 lb per sq in. The results of tests of individual cylinders tend to confirm the suggestion that concrete in the walls of Conduits 3 and 4 may be of unusual strength.

As a further indication of the quality of the concrete in the conduit entrances in general, it may be mentioned that the inspection revealed all loose or disintegrated particles of concrete to have been removed by the terrific scouring action of the water. The remaining concrete in all damaged areas was ragged and sharp but otherwise in sound condition. It is evident, therefore, that the cavitation action must have exerted terrific forces on the particles of concrete making up the conduit surfaces, particularly if consideration is given to the comparatively short total periods at which the various conduits were operated at full gate opening under heads greater than 45 ft.

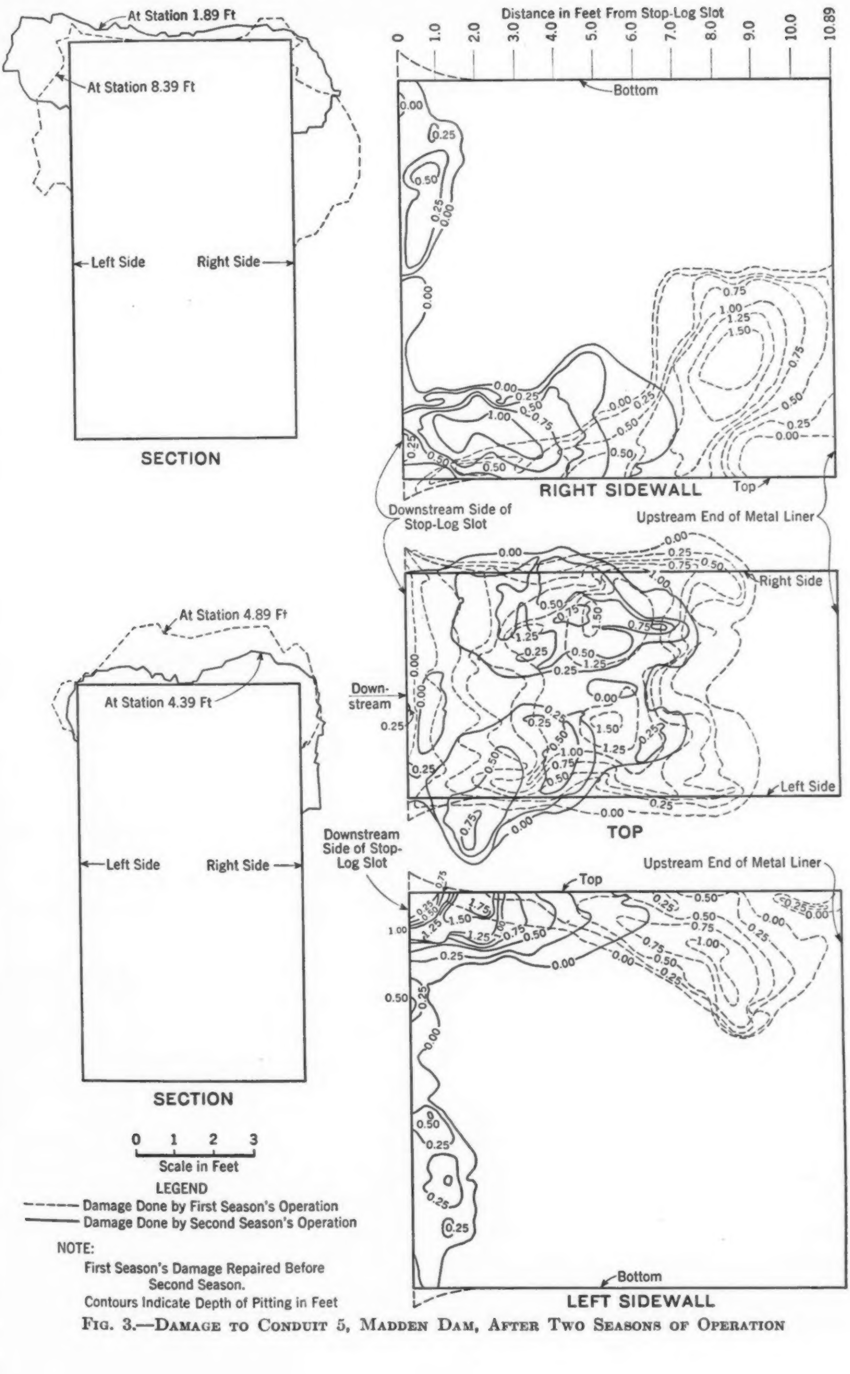


FIG. 3.—DAMAGE TO CONDUIT 5, MADDEN DAM, AFTER TWO SEASONS OF OPERATION

Attempt to Eliminate Cavitation in Madden Dam by Use of Bellmouth and Other Design Modifications

In an effort to find a solution to the problem of cavitation control, some of the conduit entrances in the Madden Dam were revised during the spring and summer of 1935 by incorporating various modifications in design. These revisions may be summarized as follows:

(*a*) Conduits 1 and 2 were reconstructed to the original entrance design, but with the addition of three air vents, 2.5 in. in diameter, each located 10.5 in. downstream from the stop-log slot, one in the center of the top of the conduit, and one in each top corner.

(*b*) The entrance to Conduit 4 was not changed, because of approaching bad weather, but a metal filler frame was installed in the stop-log slot which served to continue the entrance curve to the downstream corner of the stop-log guide. The filler contained three air vents 3 in. in diameter in the top member, one at the center and one 2 ft 10 in. each side of the center.

(*c*) Conduits 5 and 6 were reconstructed in the form of bellmouths projecting upstream from the face of the dam as indicated by Fig. 4. A metal stop-log, slot-filler frame was placed in Conduit 6 with three air vents 3 in. in diameter in the top member, one in the center and one 2 ft $1\frac{1}{2}$ in. each side of the center.

The installation of the concrete bellmouths at the entrances to Conduits 5 and 6 was completed in June, and the modification of the remaining conduits was completed during the following few weeks. Conduit 3 was not changed. The natural flow of the river was passed through the conduits until early in August when the conduits were closed and the impounding of water began.

The conduits were in use at various times during the process of filling the reservoir until the latter part of December, when a diver inspected Conduits 5 and 6 under 90 ft of water. Conduits 3, 4, 5, and 6 were later inspected in the dry when unwatered for repairs.

Conduits 1, 2, 3, and 4 had been operated at high heads for only a few hours each during the second season of operation and were found to have suffered practically no damage from cavitation during that season.

The inspection of Conduits 5 and 6 revealed that cavitation had caused considerable damage to the tops and side-walls of the conduits downstream from the stop-log slots, reinforcing bars being exposed and loose in several places. The stop-log slots themselves were in good condition, as were the surfaces of the bellmouths, except for roughening of the concrete on these surfaces immediately upstream from the stop-log slots.

The filler frame which had been fabricated of $\frac{1}{2}$-in. steel plate and placed in the stop-log slot of Conduit 6 was found to be in badly damaged condition. The bottom horizontal member was still in place with the two vertical members attached thereto. However, both vertical members had been forced downstream and downward toward the bottom of the conduit, pivoting about the bottom horizontal member of the frame as an axis. The top horizontal member of the frame was completely missing. Two of the vent pipes leading to the

filler frame were snapped off about 7 ft above the top of the frame, and the third was snapped off at the frame.

The location and extent of pitting as a result of cavitation in Conduit 5 during the second season of operation are indicated in Fig. 3. The pitting in Conduit 6 was quite similar. Superimposed on the contours showing this damage are contours indicating the damage to this conduit as a result of operation during the first season—that is, before the bellmouth entrances had been installed. It may be noted that the general proportions and locations of the pitted areas on the top of the conduit were approximately the same in the case of both seasons of operation, although the damage resulting from operation during the second season extended a slightly lesser distance into the conduit

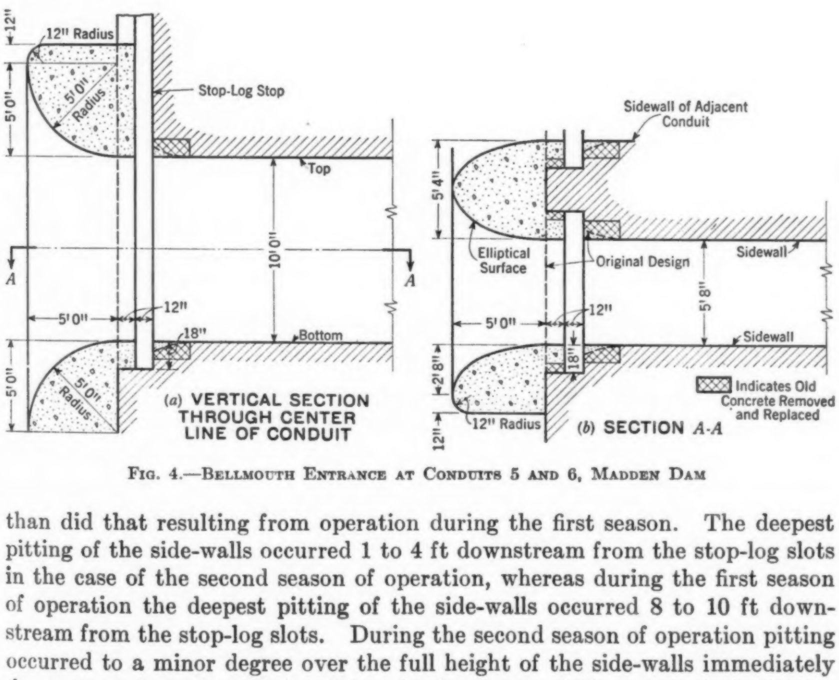


FIG. 4.—BELLMOUTH ENTRANCE AT CONDUITS 5 AND 6, MADDEN DAM

than did that resulting from operation during the first season. The deepest pitting of the side-walls occurred 1 to 4 ft downstream from the stop-log slots in the case of the second season of operation, whereas during the first season of operation the deepest pitting of the side-walls occurred 8 to 10 ft downstream from the stop-log slots. During the second season of operation pitting occurred to a minor degree over the full height of the side-walls immediately downstream from the stop-log slots. In these areas pitting was entirely absent, except near the tops of the conduits, at the close of the first season of operation, as indicated by broken lines in Fig. 3.

For comparison with Table 1 and the degree and extent of pitting resulting from operation during the respective seasons, as indicated by Fig. 3, Table 2 gives a résumé of the number of hours and minutes each conduit was operated at various heads during the second season of operation, together with the resulting damage as indicated by the maximum depth of pitting and the volume of concrete disintegrated and removed. The values given for depths of pitting correspond to the maximum-depth contours shown for the respective conduits in Fig. 3.

Nature and Causes of Cavitation

It is well known that under certain conditions of hydraulic flow a persistent void space or cavity will occur at some given location in the stream of moving water. If this cavity is not connected with the atmosphere, and therefore contains practically nothing but water vapor and flying particles or slugs of water, it is called a "cavitation pocket" and the general phenomenon of its occurrence is spoken of as "cavitation." Its existence is frequently made evident by vibration of the walls of the containing vessel and by a loud cracking sound known as "crepitation." Where a cavitation pocket occurs in contact with a solid object, such as the concrete surface of a conduit or the metal blade of a turbine runner, the solid material eventually becomes damaged by destructive disintegration commonly known as "pitting."

TABLE 2.—Madden Dam Conduits, Result of Second Season of Operation

Conduit	Hours and Minutes of Operation at Full Gate Opening: Head Range (Ft) 0–45[a] (1)	45–95 (2)	95–145 (3)	Total 45–145 (4)	Approximate Maximum Depth of Pitting (Ft): Top (8)	Right wall (9)	Left wall (10)	Volume of Concrete Removed (Cu Ft): Top (11)	Right wall (12)	Left wall (13)	Total (14)
1	2,174:28	113:31	20:30	134:01	No damage			....	....	....	....
2	2,178:44	87:57	19:58	107:55	No damage			....	....	....	....
3	3,224:52	119:00	1:35	120:35	No additional damage			....	....	....	....
4	3,189:32	122:48	1:38	124:26	No additional damage			....	....	....	....
5	1,061:07	389:08	859:35	1,248:43	0.75	1.00	1.75	12.6	9.5	7.4	30.5[b]
6	1,607:11	551:44	870:11	1,421:55	0.50	1.00	0.75	6.1	6.7	3.6	16.4

[a] 85 to 97% of this time, operation was at heads from 0 to 25 ft to pass natural river flow. [b] Includes 1.0 cu ft removed from bottom of conduit.

Cavitation pockets of the type occurring in diverging tubes are characteristically traversed from end to end by a succession of swiftly moving slugs of water separated by vapor cavities. Prof. H. E. Edgerton has taken remarkable moving pictures,[5] exposed at 3,000 per sec, showing the water slugs and vapor cavities traversing pockets of this type. One group of twenty five of these exposures[6] shows, with marked clearness, the formation, growth, and collapse of a moving vapor cavity during the one hundred and seventy fifth of a second required for it to traverse the length of the region designated as the "cavitation pocket."

Some years ago it was thought that the pitting of turbine runners, ships' propellers, and pump impellers was caused by the chemical action of nascent oxygen released in the cavitation pockets; but this theory has been discarded. It is now generally recognized that damage to materials in contact with a

[5] "Cavitation Research," by J. C. Hunsaker, *Mechanical Engineering*, Vol. 57, April, 1935, pp. 211–216.

[6] "Applied Fluid Mechanics," by M. P. O'Brien, M. Am. Soc. C. E., and G. H. Hickox, Assoc. M. Am. Soc. C. E., p. 34.

cavitation pocket is caused by the hammering or penetrating effect of extraordinarily high water pressures generated in local regions in connection with the collapse of vacuum pockets. Although to the uninitiated it may seem surprising that a moving mass of water can strike a blow sufficiently severe and concentrated, not only to disintegrate concrete but to indent the hardest metals, abundant experimental evidence proves that such action is possible and of common occurrence.

An analytical demonstration of the hammering power of water under cavitation conditions may be made in two steps: (1) Proof that the collapse of vapor cavities can produce extremely high local velocities; and (2) proof that the sudden checking of such velocities can produce extremely high local pressures. The first step may be explained most conveniently in connection with an arbitrary system in which the dimensions of the vapor cavities can be expressed by a simple formula. Consider for this purpose that Fig. 5 represents a prismatic space of square cross section with diameter d, terminated at one end by a pyramid of altitude a. First, consider this space to be bounded by solid walls, and later as representing a cavitation pocket for which three of the four boundary walls are of water. Suppose that two slugs of water of equal volume and equal original kinetic energy are traveling down this space, the one in the prismatic portion having a volume $b\ d^2$ and a velocity v_0, and the one in the

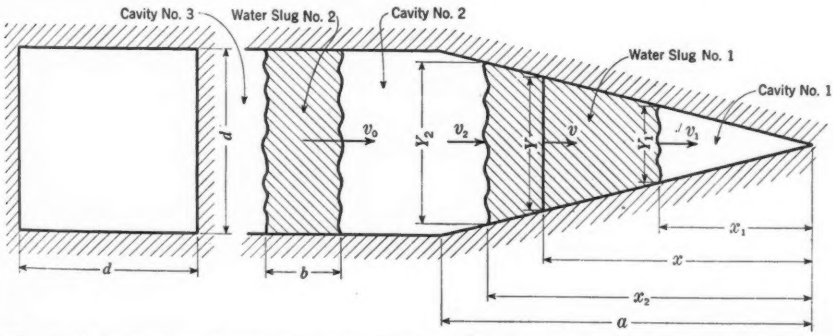


FIG. 5.—MOVEMENT OF WATER SLUG INTO THE CONFINED REGION OF DIMINISHING CROSS SECTION

pyramidal portion having the same volume but velocities of v_1 and v_2 at its forward and rear ends, respectively. The three vapor cavities adjacent to these two slugs of water are indicated in Fig. 5. If w represents the weight of a unit volume of water, the kinetic energy of a lamina of volume $y^2\ dx$ having a velocity v is

$$E_k = w\ (y^2\ dx)\ \frac{v^2}{2\ g} = w\left(\frac{x\ d}{a}\right)^2 \frac{v^2}{2\ g}\ dx \ldots\ldots\ldots\ldots\ldots (1)$$

Therefore, the kinetic energies of the two water slugs in Fig. 5 are given respectively by the first and second members of the following equation:

$$w\ b\ d^2\ \frac{v_0^2}{2\ g} = \int_{x_1}^{x_2} w\left(\frac{x\ d}{a}\right)^2 \frac{v^2}{2\ g}\ dx \ldots\ldots\ldots\ldots\ldots\ldots (2)$$

in which, in addition to symbols shown in Fig. 5: w = unit weight of water and g = acceleration of gravity.

In order to justify equating these two expressions for kinetic energy, it is necessary to neglect the dissipation of energy by hydraulic friction or viscous resistance to deformation, the storage of energy by elastic compression of the water or the conduit walls, and the release of energy by condensation of the water vapor in the cavities as the volumes of the latter decrease. This analysis thus assumes frictionless non-viscous flow, incompressible materials, and negligible vapor pressure. The effect of these assumptions will be discussed subsequently.

In Fig. 5, imagine the cross-sectional plane of abscissa x and area y^2 to consist of a weightless elastic membrane which separates the slug of water into two parts and moves forward with the velocity v. Since the volume of water must thus remain constant between the sections whose respective abscissas are x and x_1:

$$v\,y^2 = v_1\,y_1^2 \qquad (3a)$$

Since $y = \dfrac{x\,d}{a}$ and $y_1 = \dfrac{x_1\,d}{a}$, Eq. 3a becomes $v\left(\dfrac{x\,d}{a}\right)^2 = v_1\left(\dfrac{x_1\,d}{a}\right)^2$, or

$$v^2 = v_1^2\left(\frac{x_1}{x}\right)^4 \qquad (3b)$$

Inserting Eq. 3b in Eq. 2: $v_0^2\,b = \dfrac{v_1^2\,x_1^4}{a^2}\displaystyle\int_{x_1}^{x_2}\frac{dx}{x^2} = \dfrac{v_1^2\,x_1^4}{a^2}\left(\dfrac{1}{x_1} - \dfrac{1}{x_2}\right)$; or,

$$v_1 = v_0\,a\sqrt{\frac{b}{x_1^3 - \dfrac{x_1^4}{x_2}}} \qquad (4)$$

As x_1 becomes very small in comparison with x_2, Eq. 4 approaches the value: $v_1 = \dfrac{v_0\,a\,\sqrt{b}}{x_1^{1.5}}$, from which it is evident that the velocity v_1 increases without limit as x_1 becomes very small. Thus, if viscosity, compressibility, and vapor condensation did not influence the rate of movement of the water, the velocity at the tip of the slug would become infinite at the final instant of the collapse of the vapor cavity at the apex of the pyramid. It is possible also to prove by similar methods that at this same instant, and under the same assumptions, the velocity v_2 becomes zero. In other words, all of the original kinetic energy of the slug would still exist in the kinetic form; but it would be practically all concentrated into the fluid particles that make the final contact with the solid surface at the point where the last vestige of the collapsing cavity disappears, with the result that these particles would acquire enormous velocity.

It is evident that the latter statement could be proved for spaces of other shapes than the square pyramid in Fig. 5. In fact, if incompressible materials, frictionless flow, and negligible vapor pressure are assumed, the statement is perfectly general and applies to spaces of all shapes, whether bounded by solids or fluids. Under these assumptions, if a slug of moving liquid is brought to

rest by traveling into a confined vacuum space of any shape, the velocity of the last particles making the contact must become infinite.

Although the factors specifically neglected in the foregoing demonstration undoubtedly have important effects, the analysis reveals the fundamental reason why intense local concentrations of energy are associated with the collapse of vacuum cavities. From the foregoing it is apparent that if cavitation occurs in large outlet conduits of high dams there is abundant opportunity for tremendous concentrations of energy to occur at local spots along the conduit surfaces.

Under the foregoing assumptions, the pressure developed at the point where the two incompressible materials finally meet with infinite relative velocity would evidently be "super-infinite." The problem of determining the pressure developed by the collision of compressible materials moving at finite relative velocity remains to be discussed. This problem comprises the second part of the analytical demonstration of the hammering power of water, and requires a development of the proof that the sudden checking of high-water velocities can produce extremely high pressures. This proof is analogous to that which shows that high pressures in a pipe can be produced by the sudden closure of a valve. When a rapidly moving mass or slug of water strikes a rigid plane surface normal to the direction of motion, the water particles adjacent to the contact surface and near to the center of the contact area have their velocity suddenly reduced to zero and their kinetic energy suddenly changed to energy of elastic compression. By equating the values of these two kinds of energy, per pound of water, the amount of the pressure rise may be computed.

In order to make this computation accurately, it is necessary to have information regarding the compressibility of water under the unusual conditions considered. At very high pressures the volumetric coefficient of elasticity of water increases to several times the ordinary value published in textbooks.

In discussing the effects of rapid compression of water, the results of ordinary slow-compression tests cannot be used directly. In slow or isothermal tests, the heat of compression has time to be dissipated to the surrounding objects or mediums, practically uniform temperature being maintained in the specimen at the times when readings of deformation are taken. Under such conditions water at 50° F solidifies into ice at a pressure of about 110,000 lb per sq in. In rapid or adiabatic compression, the heat generated has no opportunity to escape to the surrounding objects or mediums, and therefore the temperature of the water rises as the pressure is applied, thus preventing solidification into ice.

The results of an elaborate investigation on the compressibility of water were published in 1912 by P. W. Bridgeman,[7] who presents a diagram which, when translated from metric to English units, gives the volume of one pound of water at all pressures from 0 to 180,000 lb per sq in. in combination with all temperatures from 4° to 176° F, except where the water is in the solid state. He also presents a diagram giving, for the same range of pressures and temperatures, the adiabatic rise in temperature per unit rise in pressure, as determined,

[7] "The Thermodynamic Properties of Water," by P. W. Bridgeman, *Proceedings*, Am. Academy of Arts and Sciences, Vol. 48, 1912, pp. 307–362.

not from actual quick-compression tests, but from a thermodynamic analysis of thermal measurements during slow-compression tests. By the use of these two diagrams it is possible to start with one pound of water at atmospheric pressure and any given temperature, and compute its volume and temperature after any number of successive increments of adiabatic compression. It is also possible to compute the total external work or energy contributed to the pound of water during the adiabatic compression, since the work done during any one increment of pressure is equal to the product of the mean pressure and the decrement in volume. The results of such a computation by the writers, starting with one pound of water at 50° F and extending to the limits of Bridgeman's diagrams, are given in the first eight columns of Table 3: Col. 1 gives the

TABLE 3.—COMPUTATION OF PRESSURE RISE IN WATER DUE TO DESTRUCTION OF VELOCITY BY SUDDEN IMPACT (BASED ON INITIAL WATER TEMPERATURE OF 50° F)

Pressure (lb per sq in.)	Adiabatic temperature rise (degrees F)	Temperature (degrees F)	Volume of 1 lb of water (cu in.)	Decrement in volume of 1 lb of water (cu in.)	Average pressure (lb per sq in.)	Increment of work (in-lb)	Total energy of compression per lb of water (in-lb)	VELOCITY (FT PER SEC) REQUIRED TO PRODUCE A GIVEN PRESSURE AGAINST:	
								Rigid surface	Water surface
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
0		50.0	27.68				0	0	0
	2.3			1.10	7,112	7,820			
14,223		52.3	26.58				7,820	205	237
	4.1			0.83	21,335	17,710			
28,446		56.4	25.75				25,530	370	427
	4.7			0.64	35,558	22,750			
42,669		61.1	25.11				48,280	510	589
	4.9			0.54	49,781	26,880			
56,892		66.0	24.57				75,160	636	735
	4.9			0.47	64,004	30,080			
71,115		70.9	24.10				105,240	752	869
	5.2			0.40	78,227	31,300			
85,338		76.1	23.70				136,540	856	989
	5.8			0.36	92,450	33,270			
99,561		81.9	23.34				169,810	955	1,103
	6.1			0.31	106,673	33,080			
113,784		88.0	23.03				202,890	1,043	1,205
	6.0			0.27	120,896	32,640			
128,007		94.0	22.76				235,530	1,124	1,298
	6.0			0.24	135,119	32,420			
142,230		100.0	22.52				267,950	1,199	1,385
	6.0			0.22	149,342	32,850			
156,453		106.0	22.30				300,800	1,270	1,467
	6.0			0.20	163,565	32,700			
170,676		112.0	22.10				333,500	1,338	1,545

pressure in increments of 14,223 lb per sq in. (1,000 kg per sq cm); Cols. 2 and 4 are obtained from Bridgeman's diagrams, with the values converted to English units; Col. 3 is an accumulative summation of the increments in Col. 2; Col. 5 consists of the differences between successive items in Col. 4; Col. 6 contains the averages of successive items in Col. 1; Col. 7 gives the products of the items in Cols. 5 and 6; and Col. 8 is obtained by summing the items in Col. 7.

The kinetic energy of one pound of water moving at velocity v feet per second is $12 \frac{v^2}{2g} = \left(\frac{v}{2.318}\right)^2$ in-lb. Each item in Col. 9, Table 3, gives the initial

velocity required to develop kinetic energy equal to the energy of elastic compression at the pressure indicated in Col. 1. This is found by multiplying the constant 2.318 by the square root of the corresponding item in Col. 8. In other words, any pressure value in Col. 1 is that generated when water moving at the velocity given by the corresponding figure in Col. 9 strikes a rigid plane surface normal to its path. For example, if water is moving at 1,338 ft per sec, its impact on such a surface will develop a pressure of 170,676 lb per sq in. Under ordinary conditions, a velocity as high as 1,338 ft per sec could be acquired only in the case of flow into a converging space, according to principles previously explained. Thus, if the aforementioned rigid plane surface were in a position to block the end of such a converging space, the pressure of 170,676 lb per sq in. would be developed against its face. However, if the surface were not a true plane, but contained crevices, depressions, or intracrystalline cracks such as to cause small portions of the water to acquire velocities much higher than the general velocity of 1,338 ft per sec before being finally brought to rest, local pressures much higher than the general pressure of 170,676 lb per sq in. would be developed.

The law governing the foregoing process of pressure multiplication undoubtedly changes when the crevices become so small as to approach molecular dimensions. However, what has been given contains a suggestion as to the source of pressures known to be sufficient to penetrate or deform the hardest metals.

In the foregoing analysis, when the slug of water hits the rigid surface, a pressure wave of intensity p is generated at the contact surface and travels out into the liquid at the acoustic velocity

$$u = \sqrt{\frac{g\,e}{w}} \qquad (5)$$

the phenomenon being similar to the generation of a water-hammer wave in a pipe by the sudden closure of a valve. However, when the slug of water strikes a water surface normal to the direction of motion, two pressure waves travel out from the contact surface in opposite directions, each moving at the acoustic velocity. This action may be described in more detail by reference to Fig. 6, which shows a slug of water moving at velocity v and striking a water surface. Consider within the slug a rectangular prism of water of cross-sectional area $(dx)^2$, with its sides normal to the contact surface. The enlarged section of a portion of this prism in Fig. 6 indicates that the contact surface is being forced down with a velocity v_c. At an instant dt seconds after contact, the row of particles which were originally in surface $C'C'$ have moved down to CC, and the particles which were in $A'A'$ at the instant of contact have moved down to AA. A pressure wave front having the acoustic velocity u relative to the surrounding fluid has traveled upward from $C'C'$ to AA, and another has moved downward from $C'C'$ to BB. The shaded volume is under a pressure p and has a unit weight w and a coefficient of elasticity E. The volume corresponding to the unshaded area is still under its original pressure. Since the force required to retard or accelerate a mass of water is equal to the mass of water multiplied

by the rate of change in velocity: Force on CC = (Mass of $A'A'C'C'$) the rate of reduction in its velocity = (Mass of $C'C'BB$) the rate of increase in its velocity; or

$$p\,(dx)^2 = \left(\frac{w}{g}\,u\,dt\right)(dx)^2\left(\frac{v - v_c}{dt}\right) = \left(\frac{w}{g}\,u\,dt\right)(dx)^2\left(\frac{v_c}{dt}\right) \ldots\ldots (6)$$

Therefore, $v_c = 0.5\,v$.

In other words, the velocity of particles at the contact surface is one half the initial velocity of the slug of water; therefore, one fourth of the original kinetic energy of these moving water particles is still in the kinetic form, and

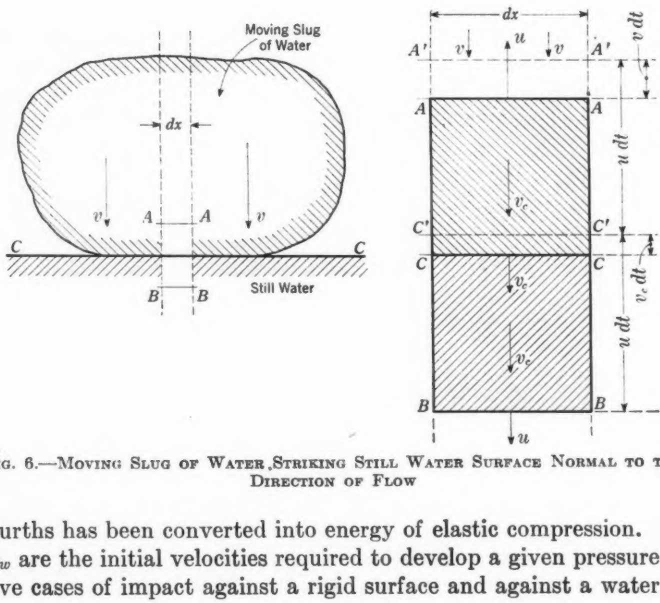


FIG. 6.—MOVING SLUG OF WATER STRIKING STILL WATER SURFACE NORMAL TO THE DIRECTION OF FLOW

three fourths has been converted into energy of elastic compression. Thus, if v_r and v_w are the initial velocities required to develop a given pressure p in the respective cases of impact against a rigid surface and against a water surface:

$$\frac{3}{4}\,\frac{(v_w)^2}{2\,g} = \frac{v_r^{\,2}}{2\,g} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

and from this it is seen that $v_w = v_r\sqrt{\frac{4}{3}} = 1.155\,v_r$. In other words, a velocity 15.5% higher is required to produce a given pressure if the impact is on a water surface than if it is on a rigid surface. Col. 10 of Table 3 is computed on the basis of this principle.

From the foregoing it is seen that when a cavity in water collapses, the pressure developed at the final point of contact may be very high if this point is in contact with a solid surface and partly surrounded by water, although not quite as high as if it were fully surrounded by solid materials.

The familiar effects of water hammering sufficiently intense to indent and disintegrate the surfaces of hard metals furnish abundant proof that pressures

approaching or exceeding the higher values of Table 3 actually occur in engineering machines and structures.

In the case of metal surfaces the development of deep pitting is a slow process, usually requiring months of continuous operation. The long-continued intense hammering on tiny local spots overstresses the surface of the metal until fatigue cracks appear. The enormous pressure developed in such cracks, in accordance with the foregoing theory, then quickly enlarges and spreads them. In the case of concrete surfaces slightly permeable to water, the penetration of enormous pressures into the pore spaces easily breaks the bond between the aggregate particles, so that extremely destructive effects can occur within comparatively few hours.

Description of Apparatus for Cavitation Tests

Cavitation may occur at any point in a hydraulic system when the pressure at that point is reduced to the vapor pressure of the liquid flowing in the system. This requirement for the cavitation phenomenon to occur, therefore, precludes the study of its possible occurrence in hydraulic structures by means of small undistorted models of the usual laboratory type in which air at ordinary barometric pressure and water at room temperature are used and relative headwater and tailwater depths are represented to scale. In such a model, the whole atmospheric pressure is effective on the free surfaces of the liquid used in the model and is transmitted throughout that liquid so that the absolute pressure in the regions of the model corresponding to the cavitation regions of the prototype is far greater than the vapor pressure of the liquid in the model. In order that the basic conditions which produce cavitation in certain regions of the prototype may be correctly represented in the model, it is necessary that the pressures in these regions of the model be reduced to the vapor pressure of the liquid used in the model.

In planning the cavitation tests of the conduits at Tygart River and Madden dams, modification of the vapor pressure of the liquid by thermal or chemical means was eliminated from consideration because of its inconvenience from the standpoint of laboratory practice. Therefore, two possible means of simulating the basic conditions which produce cavitation in the critical regions of the prototype presented themselves—namely: (1) The design of a testing apparatus in which absolute pressure could be reduced in such a way as to simulate reduction of the atmospheric pressure on the headwater and tailwater surfaces in the model; and (2) the design of a testing apparatus such that, by distorting the headwater pressure and the discharge scale, the correct cavitation potentiality would be produced in the critical area of the conduit. The first of these testing facilities was used exclusively in the earlier tests, but for the later tests the second was also used, particularly to facilitate tests involving the admission of air into the conduits to alleviate or cushion the shock of cavitation. For the purpose of this paper these two testing devices are designated "enclosed-tank cavitation apparatus" and "diverging-tube cavitation apparatus," respectively.

Enclosed-Tank Cavitation Apparatus.—A general understanding of the design of the enclosed-tank cavitation apparatus may be had from Fig. 7.

In testing models in this apparatus, the absolute pressures on the surfaces corresponding to the headwater and tailwater surfaces in the prototype had to be reduced in almost direct proportion to the model scale. Consequently, the entire fluid medium had to be subjected to subatmospheric pressure, thus necessitating that the apparatus constitute an enclosed system in which the water supply was continuously circulated. As indicated by Fig. 7, the apparatus consisted essentially of a head tank (*A*), 32 in. in diameter and flattened on the top at the outlet end (*B*), where it was welded to a rectangular section (*C*) containing the model, the outlet end of the rectangular section in turn

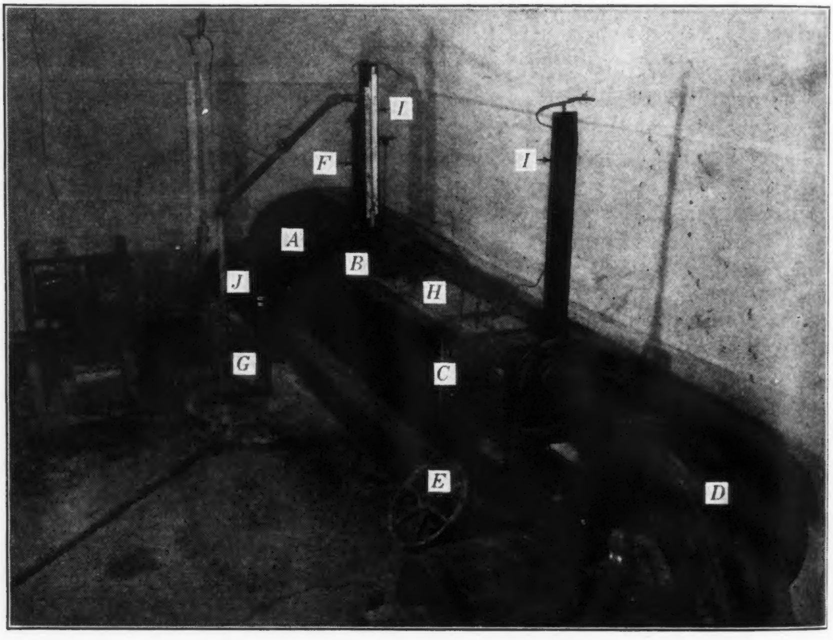


FIG. 7.—ENCLOSED-TANK CAVITATION APPARATUS

being connected by an 8-in. pipe line through a centrifugal pump (*D*) and control valve (*E*) to the inlet end of the head tank. An air trap (*F*) was placed in the top of the head tank and connected to a vacuum pump (*G*) which supplied the necessary reduction in pressure to the system of the testing apparatus. An opening was placed in the top of the rectangular section of the apparatus to facilitate the insertion and removal of models. A one-inch plate of annealed glass (*H*) served as a cover for the opening and also as an observation window to facilitate study of flow conditions and the action of cavitation in the model. The glass plate was protected against direct contact with the concrete of the model by a rubber gasket and was sealed against leakage by an application of sculptor's plastic waterproof clay between the edge of the glass plate and the edge of the opening in the tank. For tests involving measure-

ments of the pressure distribution over a longitudinal section of the conduit, the annealed plate glass cover was replaced by a brass plate containing a network of piezometer openings. Mercury gages (*I*) of the cistern manometer type were provided at each end of the model to determine pressures at those points (corresponding to points H' and R' of Fig. 8). Baffles were provided

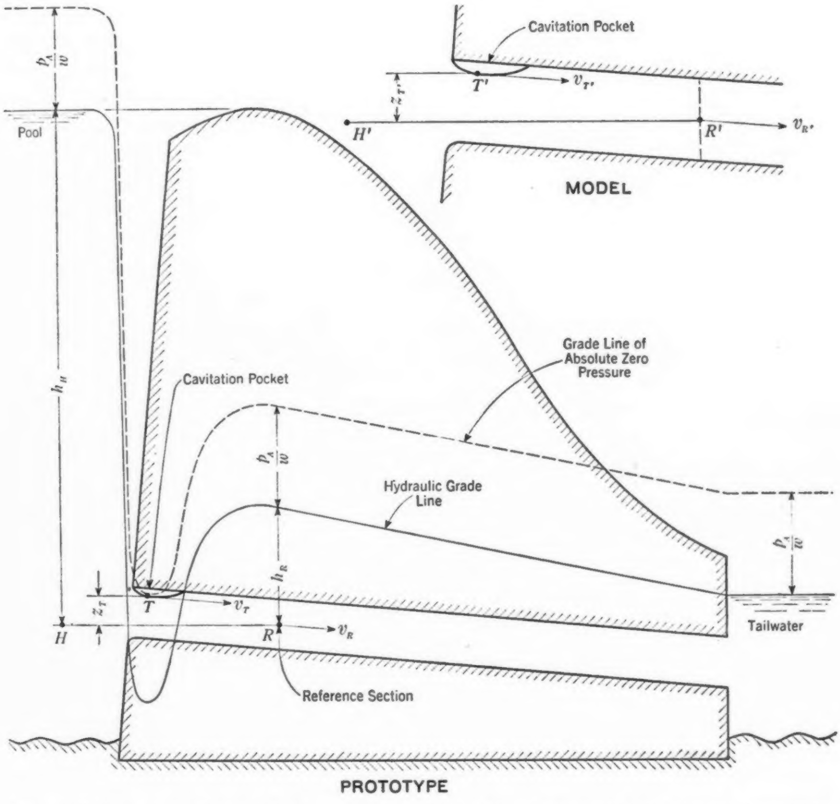


FIG. 8.—TYPICAL DAM SECTION SHOWING PRINCIPLES OF MODEL OPERATION TO SIMULATE CAVITATION

at the inlet end of the head tank to eliminate excessive turbulence in the water and to provide good approach conditions to the model. Provision for measuring the discharge through the model was made by a pitot tube (*J*) in the 8-in. pipe line, the pitot-tube installation having been calibrated by traversing the cross section of the pipe.

The models tested in this apparatus were generally constructed of concrete, with such appurtenances as stop-log slots and guides constructed of wood or sheet metal. In tests to study pitting, inserts of a specially designed concrete of low strength (see section "Experiments to Find a Material to Simulate Pitting") were cast in the critical regions of the model.

To facilitate observation and study, it was necessary with this testing apparatus to construct and test models representing only one half of the prototype cross section; that is, the prototype was represented in the model as halved by a diametrical plane, the inclination of the plane being chosen to represent the prototype to the best advantage for the purpose of the particular test.

In placing the models in the testing apparatus, the diametrical plane was coincident with the bottom surface of the glass (or brass) cover plate for the rectangular section of the apparatus. In many cases, therefore, it was necessary to represent the prototype in a revolved position—that is, with the conduit inverted or turned on its side. However, tests of models of the same conduit, shown in various revolved positions, indicated that such orientation had no noticeable effect on the results of the tests.

In conducting tests in this apparatus, it was found necessary to operate for several minutes to exhaust air entrapped and dissolved in the water of the system before actual test results could be obtained. For this reason, it was found impracticable to conduct tests, with this apparatus, that required the admission of air into the model to alleviate or cushion the shock of cavitation. The diverging-tube cavitation apparatus was designed to facilitate such tests.

Diverging-Tube Cavitation Apparatus.—A general understanding of the design of this apparatus may be had from Fig. 9. The horizontal pipes along the wall behind the gage boards are not a part of the testing equipment. As indicated, the apparatus consisted essentially of a head tank (*A*) 15 in. in diameter, to the outlet end of which was connected the model to be tested (*B*), constructed of a pyroxylin material and representing the full cross section of the prototype. The outlet end of the model in turn was connected to a diverging tube (*C*) which led to the laboratory sump. The transparent nature of the pyroxylin material made unnecessary any orientation of the conduit away from its normal position. Since at the outlet end of the diverging tube the pressure was atmospheric and the velocity of flow relatively low, this tube served, through the smaller cross section and higher velocity at its throat, to provide sub-atmospheric pressure at the outlet end of the model. It was possible by means of this apparatus to test models at headwater pressures greatly beyond those represented by direct scale reductions of the ones in the prototype, with the result that velocities were correspondingly exaggerated. Such exaggeration of the head and velocity scales was necessary in order that absolute pressures in the critical regions of the conduit be in correct relation to the absolute pressures which would exist in the prototype. Provision for measuring the pressures at each end of the model (corresponding to points H' and R' of Fig. 8) was made by mercury gages (*D*) of the U-tube type.

Connection of the gage to the lower end of the model was by means of a piezometer opening on each of the four sides of the conduit, the individual openings being connected through a common header to a single gage. An air cushion consisting of a glass jar (*E*) was provided in the line connecting the piezometer opening in the head tank with the mercury gage for determining the pressure at that point. Suitable baffles were provided in the head tank to eliminate excessive turbulence and to provide smooth approach conditions

to the model. The rate of discharge of water through the model was measured by a venturi meter in the pipe line leading from the pump.

In operating this testing apparatus, water was drawn from the laboratory sump and returned to it, and therefore the elimination of entrapped and dissolved air in the system was greatly simplified. It was possible, thus, to study the effect on cavitation of venting the conduits without encountering the difficulty of having the air that was admitted to the model reappear in the head tank. To facilitate the tests of venting the conduits, provision was made for introducing air into the model at the inlet end of the conduit (*F*) and also immediately below both the emergency (*G*) and service (*H*) slide-gate slots However, since the emergency slide gates would be operated only on rare

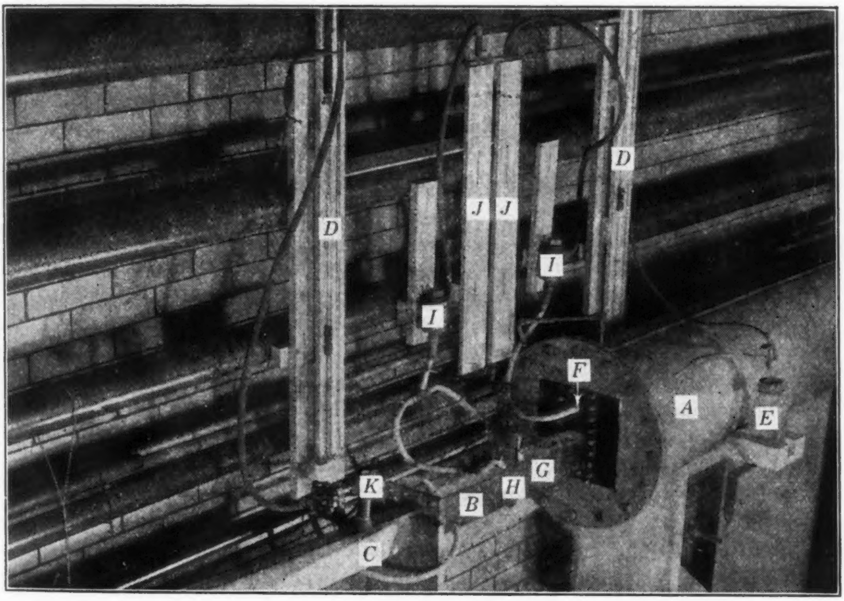


FIG. 9.—DIVERGING-TUBE CAVITATION APPARATUS

occasions, only the service slide gate was simulated in the model; but slots for both were included. The rate of flow of air into the model during venting tests was measured by means of circular, round-edged, brass orifices (*I*) provided with U-tube water manometers (*J*) to measure the head loss. The valve (*K*) in the upper end of the diverging tube served as a bleeder to adjust the pressure at the lower end of the model.

NOTATION

The notation adopted in this paper conforms essentially with American Standards Symbols for Hydraulics[8] compiled by the American Standards Association with Society representation, and approved by the Association in 1929.

[8] ASA—Z10*b*—1929.

Frequent mention will be made of three points whose locations in the prototype are indicated in Fig. 8. The point R is at the center of a reference cross section of the conduit, H is in the quiet water of the headwater reservoir at the same elevation as R, and T is on the streamline portion of the boundary of any given cavitation pocket. The points in the model corresponding geometrically to R and T are designated as R' and T', whereas a point in the quiet water of the headwater tank at the same elevation as R' is designated as H'. The elevation of T above R is designated as z_T and the elevation of T' above R' is designated as z_T'. It is to be noted that if the orientation of the model with respect to the direction of gravity does not duplicate that of the prototype, the ratio of z_T' to z_T will not necessarily be equal to the linear scale L used in constructing the model.

A small or lower-case letter is used to designate the general value of any given quantity pertaining to the prototype structure or water, and the same letter primed is used to denote the general value of the corresponding quantity pertaining to a model of this structure or the liquid used in this model. The same capital letter is used to denote the corresponding scale ratio. For example: If l designates the length of any line in the prototype, then l' designates the length of the corresponding line in the model, and L denotes the linear scale ratio $\frac{l'}{l}$. The value of a quantity at any particular point in the prototype or model is designated by using the letter denoting that point as a subscript on the lower-case letter denoting the general value of that quantity in the prototype. For example: p_R' denotes the pressure at the center of the reference section in the model conduit.

All models described herein are geometrically similar to the prototype, in so far as their solid parts are concerned, and therefore have a definite linear scale L. However, "gravitational distortion of scale" occurs when the orientation of the model with respect to the direction of gravity does not duplicate that of the prototype, and "operational distortion of scale" occurs when the absolute pressures due to headwater and tailwater are not reduced in exact accordance with the linear scale L, or where the liquid in the model is not chilled to reduce its absolute vapor pressure in exact accordance with the same scale. The final formulas of the following paragraphs are written to include cases where these distortions are used.

In accordance with Torricelli's theorem, the velocity of flow in the prototype at point T is given by

$$v_T = c_T \sqrt{2\,g\left(\frac{p_H}{w} - z_T - \frac{p_T}{w}\right)} \qquad \text{(8a)}$$

and the velocity at point T' in the model is given by

$$v_T' = c_T' \sqrt{2\,g\left(\frac{p_H'}{w} - z_T' - \frac{p_T'}{w}\right)} \qquad \text{(8b)}$$

in which: v_T = the velocity of a streamline filament at point T; c_T = coefficient of velocity for flow from point H to point T written to give the filament velocity

v_T; and z_T = the elevation of point T above point R. At first it will be assumed: (*a*) That the model is built to a definite linear scale L; (*b*) that it is set in the apparatus without gravitational distortion; (*c*) that the headwater and tailwater depths are adjusted to the linear scale; (*d*) that the water in the model is chilled until its vapor pressure is reduced in the same scale; and (*e*) that the atmospheric pressure on the headwater and tailwater surfaces is reduced in the same scale. If these conditions are satisfied, Eq. 8*b* can be written in the form,

$$V\, v_T = c_T' \sqrt{2\, g \left(\frac{L\, p_H}{w} - L\, z_T - \frac{L\, p_T}{w}\right)} \quad \ldots\ldots\ldots\ldots (8c)$$

in which: V = scale ratio, for velocity $= \frac{v'}{v}$; L = scale ratio for length $= \frac{l'}{l}$; p_T = absolute pressure at point T, or vapor pressure of water at the given temperature; and p_H = absolute pressure at point H, given by

$$p_H = w\, h_H + p_A$$

in which: h_H = head of water above point H, p_A = absolute pressure of air on the free surfaces of the liquid, and w = unit weight of the liquid. If it is assumed that the roughness of the model surface has been adjusted until $c_T' = c_T$ and that the values of g and w are alike in the model and prototype, the expression, $V = \sqrt{L}$ (known as Froude's law) may be obtained by dividing the two members of Eq. 8*c* by the corresponding members of Eq. 8*a*. This result means that if the model is operated according to Froude's law (that is, with all the aforementioned adjustments) the velocity at any point in the model will become equal to that of the corresponding point in the prototype multiplied by the square root of the linear scale, and the absolute pressure at the point T' will become equal to the vapor pressure of the water used in the model. If this is true for all points in the model corresponding to points on the boundary of the cavitation pocket in the prototype, a cavitation pocket must open up in the model, and will correspond exactly to the prototype pocket in shape, size, and location.

In the foregoing it was assumed that the coefficient of velocity c_T' for flow from H' to T' could be considered equal to c_T. For pockets near the conduit entrance where the friction losses are small in comparison with the total head, this assumption is not likely to be appreciably in error. For pockets farther from the inlet to the conduit, where the friction losses are larger, the problem of adjusting the roughness of the model surfaces to make c_T' equal to c_T presents difficulties if a high-precision solution is desired. Similar difficulties are encountered in all hydraulic model systems where the effects of gravity and viscosity are both important, and therefore require the satisfaction of the two mutually conflicting laws of Froude and Reynolds. Fortunately, a highly precise duplication of this coefficient is not required in typical cavitation studies, since a design which approaches even near the cavitation limit may be classed as unsatisfactory.

In the aforementioned method of model operation in accordance with Froude's law, the absolute pressure in the model not only at T' but at every

other point in the liquid is equal to the absolute pressure at the corresponding point in the prototype multiplied by the linear scale ratio L. In spite of the simplicity of this method of operation, numerous occasions arise for deviating from it in conducting miscellaneous cavitation studies by means of models. These occasions are usually created by the limitations of the available apparatus. Because of the location of an observation window, a conduit of rectangular cross section in the prototype may have to be inverted or laid on its side in the model; or because of the inconvenience of chilling the water in the model to reduce its vapor pressure, a slight variation from the linear-scale representation of the atmospheric pressure may be made; or the suction apparatus may not develop sufficiently low pressure to permit undistorted scale testing. When such departures are made from operation in accordance with a single linear scale for all dimensions and pressure heads, the absolute pressures are no longer to scale over the entire region occupied by the liquid in the model. This means that a group of several cavitation pockets in the model cannot be studied simultaneously. Attention must be focused on a limited region in which the occurrence of a cavitation pocket is known or suspected; and the general pressure and flow conditions of the model must be adjusted to produce the correct cavitation potentiality in that limited region. A readjustment of the general conditions must be made when a pocket in another location is studied. The equations for making these adjustments are given subsequently.

Referring to Fig. 8, it is assumed that the "reference section" R in the prototype corresponds to the location of an accurate mercury piezometer in the model. In most of the cavitation tests at Pittsburgh, the models did not represent the entire conduit, but only the upstream portion, and the piezometer was placed near the downstream end of the model. In the application of these formulas it is assumed that, for a given water elevation in the reservoir, either the mean velocity V_R or the pressure head h_R at the prototype reference section is known by computation or by actual experiments on the prototype. For reasons already mentioned, a direct determination of both of these quantities by model studies is not feasible, because the effects of gravity and viscosity are both involved to an important degree.

Referring again to Fig. 8, the mean velocity at the prototype reference section may be expressed by:

$$v_R = c_R \sqrt{2\,g\left(\frac{p_H}{w} - \frac{p_R}{w}\right)} \qquad \text{(9a)}$$

and the corresponding mean velocity in the model by:

$$v_R' = c_R' \sqrt{2\,g\left(\frac{p_H'}{w} - \frac{p_R'}{w}\right)} \qquad \text{(9b)}$$

in which: v_R = mean velocity in a conduit cross section at point R; c_R = coefficient of velocity for flow from point H to point R, written to give the mean velocity v_R; and p_R = absolute pressure at point R, equals

$$p_R = w\,h_R + p_A$$

in which: h_R = pressure head at point R. By assuming that the model rough-

ness is adjusted so that c_R' and c_R are equal and eliminating them between these equations, considering again that g and w are alike in model and prototype, there is obtained:

$$\frac{v_R'}{v_R} = \sqrt{\frac{p_H' - p_R'}{p_H - p_R}} \quad \text{(9c)}$$

If m represents the ratio of v_T to v_R, Eqs. 8a and 8b may be expressed in the form:

$$m\, v_R = c_T \sqrt{2\, g \left(\frac{p_H}{w} - z_T - \frac{p_T}{w}\right)} \quad \text{(10a)}$$

and

$$m'\, v_R' = c_T' \sqrt{2\, g \left(\frac{p_H'}{w} - z_T' - \frac{p_T'}{w}\right)} \quad \text{(10b)}$$

Considering that m and m' must be identical if the flow patterns for prototype and model are to be geometrically similar, and assuming as before that c_T' and c_T are made identical by proper adjustment of the model roughness, and eliminating these quantities from Eqs. 9 and 10, there results:

$$\frac{v_R'}{v_R} = \sqrt{\frac{p_H' - w\, z_T' - p_T'}{p_H - w\, z_T - p_T}} \quad \text{(11)}$$

Eqs. 9c and 11 may be combined in various ways to determine the proper methods of model operation to develop a given pressure p_T' at any given location in a model, such as point T' in Fig. 8. If this value of p_T' is the vapor pressure corresponding to the temperature of the water in the model, and if point T is on the boundary of a cavitation pocket in the prototype, then point T' must also be on the boundary of a corresponding cavitation pocket in the model.

Suppose that the values of p_H, p_R, z_T, and p_T are known for the prototype, that cavitation in the model is to be investigated in a limited region for which z_T' is known, that the vapor pressure p_T' for the liquid of the model is known, and that by means of the vacuum pump or other suction device a known low absolute pressure p_R' can be maintained at the reference section in the model. To operate the model properly, it would be required to determine the absolute pressure p_H' which must be carried in the model headwater tank at the point H'. The desired value may be obtained by eliminating $\frac{v_R'}{v_R}$ in Eqs. 9c and 11, resulting in the following equation:

$$p_H' = \frac{p_R'\,(p_H - w\, z_T - p_T) - (w\, z_T' + p_T')\,(p_H - p_R)}{p_R - w\, z_T - p_T} \quad \text{(12)}$$

If the model is correctly built and its surface roughness correctly adjusted, simultaneous setting of the two control piezometers at the values of p_H' and p_R' prescribed by Eq. 12 will automatically cause the velocity in the conduits to assume the value v_R', and the pressure at point T' to drop to the vapor pressure p_T', thereby causing the model cavitation pocket to appear in its proper form and location.

If it is desired to regulate the model by controlling the discharge with the aid of a valve and a venturi meter or other meter in the supply pipe, instead of regulating the pressure at point H' in the model head tank (the known quantities being the same as in the preceding paragraph except that v_R is given instead of p_H), the equation for determining the necessary model conduit velocity v_R' may be found by eliminating p_H' between Eqs. 9c and 11, resulting in the following:

$$v_R' = v_R \sqrt{\frac{p_R' - w\, z_T' - p_T'}{p_R - w\, z_T - p_T}} \qquad (13)$$

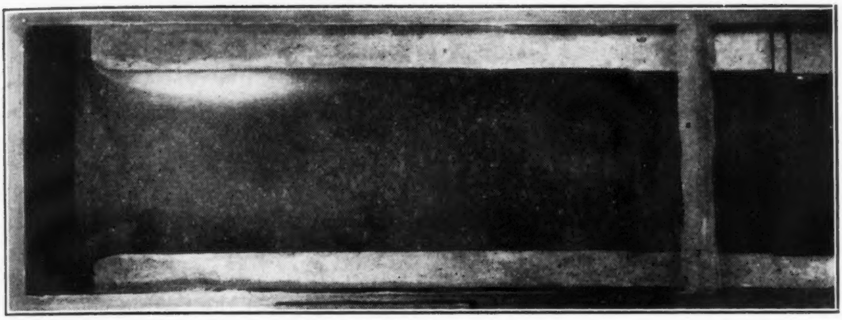
If the model is correctly built and adjusted for roughness, the setting of v_R' and p_R' at the values prescribed by Eq. 13 should give exactly the same flow conditions as those described in the preceding paragraph. Actually, however, due to natural and unavoidable errors in the meters and gages and to small imperfections in the model and its adjustment, minor discrepancies are likely to occur in the results of the two methods of operation. A perfect roughness adjustment is not usually feasible in actual laboratory practice, as the highest polish which can be produced on the available materials is insufficiently smooth.

In connection with the discussion of principles of model operation, mention may be made of the comparative advantages of the diverging-tube and enclosed-tank types of apparatus. The relative merits of the former are as follows: (1) It is much cheaper, provided that a good-sized pump or other source of fairly liberal flow under moderate head is already available; (2) its operation is much simpler, quicker, and more convenient, since it dispenses with the procedure for freeing the tank from entrained air; (3) it lends itself to studies of problems of cavitation mitigation by air admission; and (4) its higher throat velocity is advantageous in conducting actual pitting tests on models. The principal disadvantage of this apparatus is that feature which limits the conduit velocity to values greater than the spouting velocity of water at a head equivalent to atmospheric pressure. Unless a liberal flow of water is available, this requires that the models be quite small. Furthermore, unless an enclosed water storage tank is provided, this apparatus cannot be used advantageously in studies in which close control of the air content of the water is required. The principal feature of the enclosed-tank apparatus which justifies its use, in spite of its greater cost and lesser convenience, is that it permits the use of much lower conduit velocities. For a pump of given discharge capacity, a lowering of conduit velocity results in an increase in the model conduit cross section, thus allowing the use of larger models.

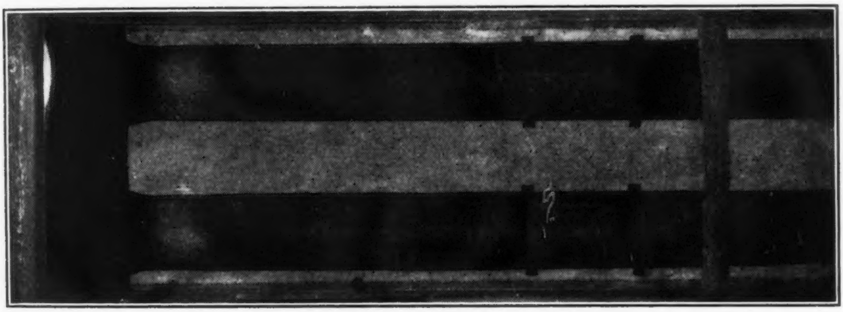
A third type of apparatus could also be built, in which the necessary degree of vacuum would be obtained by a vertical non-flaring draft tube about 34 ft high, but this was not feasible in the laboratory.

Computation of Gage Settings for Cavitation Tests on a Model Conduit

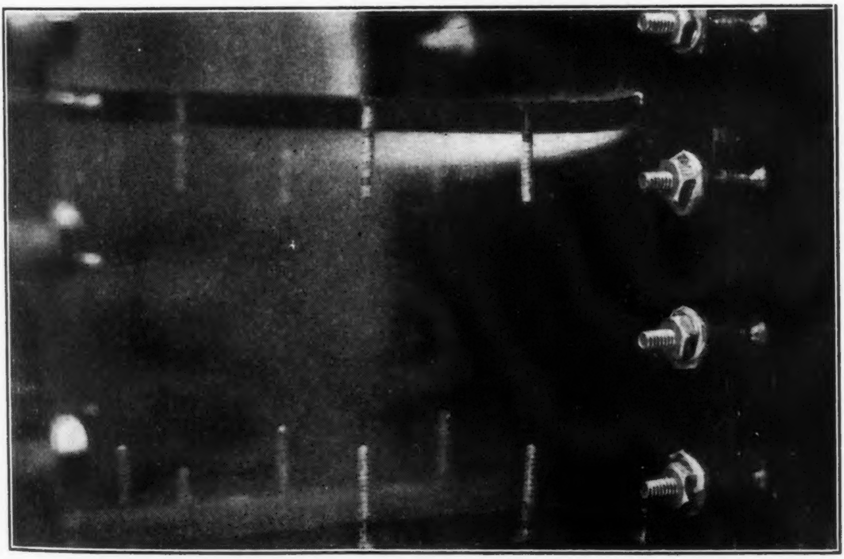
Models of various conduit entrances on several different scales have been built and tested at the laboratory in Pittsburgh in either the enclosed-tank apparatus or the diverging-tube apparatus. The first step in making such a



(a) Model Scale, 1 : 15, Enclosed-Tank Apparatus



(b) Model Scale, 1 : 20, Enclosed-Tank Apparatus



(c) Model Scale, 1 : 40, Diverging Tube Apparatus

FIG. 10.—ORIGINAL CONDUIT ENTRANCE, MADDEN DAM, OPERATING AT A 168-FT HEAD

test is to compute the necessary settings of the control gages. In each apparatus there are three of these gages: (1) The headwater pool gage, (2) the conduit reference-section gage, and (3) the gage on the venturi meter, pitot tube, or other flow meter in the supply pipe. If the prototype quantities p_H, p_R, and v_R are known, the setting for any one of these gages may be assumed, and the settings for the other two computed from them by Eqs. 11 and 12. However, the assumption must be such as to make all three settings fall within the capacity of the apparatus. The various models tested were geometrically similar to the prototype in their solid portions. However, in practically all the tests in the enclosed-tank apparatus, "gravitational distortion" was introduced by tipping or inverting the models in the testing apparatus; and in the tests with both types of apparatus "operational distortion" was introduced by omitting to chill the liquid until its vapor tension was reduced in proportion to the linear scale, or by the arbitrary choice of one of the three aforementioned gage settings. Under such conditions, as previously explained, the pressure scale would not be constant at all points in the model liquid, and the same is true of the velocity scale. The adjustment of gage settings was made to give the correct pressure condition only in a small local region containing the upstream boundary of a single known or suspected cavitation pocket.

The following example shows a typical computation for testing a 1 : 20 scale model of a pair of conduits of the Madden Dam in the enclosed-tank apparatus. Distortion of both of these types is used. A photograph of this model is reproduced in Fig. 10(*b*). The model conduits are split along the horizontal plane passing through their axes, the upper halves of the conduits being reproduced in the model and tested in the inverted position, and the top of the prototype conduit being represented in the bottom of the model. The purpose of the inversion is to make visible the large cavitation pocket known to form against the top of the conduit just inside the entrance. The computation establishes the gage settings necessary to develop the pressure corresponding to the vapor pressure of the water in this critical region of the model. The dimensions of the model were chosen to utilize most of the capacity of the pump when operating under the condition corresponding to full head on the conduits. In this example a pump discharge of 2.343 cu ft per sec is assumed. This fixes the necessary setting of the flow-meter gage, leaving the readings of the other two gages to be computed. The prototype dimensions required in the computation are shown in Fig. 11; the headwater is assumed at El. 204.7. This value of headwater elevation is used in the example since it represents the greatest head under which the conduit discharge has been verified by actual discharge measurements on the prototype. While conducting the tests on the Madden models, the feasibility of including certain refinements in the settings of the apparatus did not become apparent until near the close of the work. For this reason the computation procedure in the example is not exactly the same as that used in the actual tests.

The assumptions used in this example may be listed as follows:

Applying in General.—The headwater elevation is 204.7 ft and the tailwater elevation is at a level below the bottom of the conduit; water temperatures, in degrees Fahrenheit, are, for the prototype, 77, and for the model, 68; and the

barometric pressures, in inches of mercury, are, for the prototype, 29.500, and in the laboratory at the time of the model tests, 29.655. During the test the discharge setting of the pump is 2.343 cu ft per sec; and the rating equation for the prototype conduit (based on pitot-tube tests) is

$$v_R = 7.06 \sqrt{h_H} \qquad \text{(14)}$$

The model setting was such as to produce the correct cavitation potentiality at the top of the conduit, just inside the entrance.

Applying to the Prototype.—The total head on the conduit, h_H, is 110.0 ft (see Fig. 11); and the corresponding velocity v_R at the conduit reference section

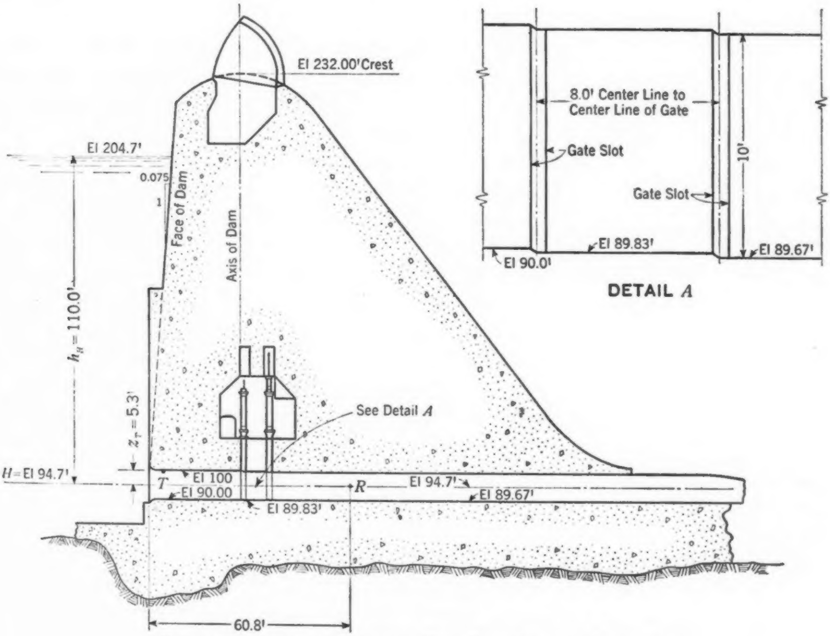


FIG. 11.—SECTION OF MADDEN DAM SPILLWAY AND CONDUIT

is $7.06 \sqrt{110.0} = 74.0$ ft per sec. Since the water temperature is 77° F, the vapor pressure p_T is 56 lb per sq ft; and, for a barometric pressure of 29.500 in. Hg, the atmospheric pressure p_A equals $\frac{29.500}{12}(13.59 \times 62.4) = 2{,}085$ lb per sq ft. Since the elevation of point T, Fig. 11, is 100.00, the value of $z_T = 100 - 94.7 = 5.3$ ft. The absolute pressure at point H is $p_H = w\, h_H + p_A = 62.4 \times 110.0 + 2{,}085 = 8{,}949$ lb per sq ft.

Applying to the Model.—The linear scale ratio $L = \frac{l'}{l} = \frac{1}{20}$. With the axis of the conduit horizontal, the model is inverted in the apparatus; therefore, $z_T' = \frac{-5.3}{20} = -0.26$ ft. For a water temperature of 68° F, $p_T' = 48.8$ lb

per sq ft; and corresponding to a barometric reading of 29.655 in. Hg, p_A' = 2,096 lb per sq ft. With the cross-sectional area of the two half conduits equal to 0.1417 sq ft, the velocity $v_R' = \frac{2.343}{0.1417} = 16.53$ ft per sec. Finally, the absolute pressure required in the headwater tank at point H' (at the level of the glass window) may be obtained from Eq. 11, thus:

$$\left(\frac{16.53}{74.0}\right)^2 = \frac{p_H' - 62.4 \times - 0.26 + 48.8}{8{,}949 - 62.4 \times 5.3 - 56}$$

from which $p_H' = 464.8$ lb per sq ft. This corresponds to a vacuum of 2,096 − 464.8 = 1,631.2 lb per sq ft, or 11.35 lb per sq in. less than the atmosphere of the room.

In the enclosed-tank apparatus the gage on the headwater tank is of the mercury-cistern type, having a rating formula to give the pressure in the tank at the level of the glass window. By this formula the gage setting necessary to give the foregoing value of p_H' is readily found.

After setting the model in the enclosed-tank apparatus, sealing the glass observation plate (window) in place with waterproof plastic clay, and filling the apparatus with water, it is necessary to remove as much of the entrained and dissolved air as possible before attempting to make accurate settings of the headwater mercury gage and of the flow-meter gage. This is done by applying the vacuum gradually, while slowly circulating the water in the apparatus, and occasionally stopping the flow by the main valve in the pipe line. This permits the bubbles of entrained air to collect in the air dome, from which they are removed by the vacuum pump. Observation through the glass observation plate (window) indicates when the water no longer contains visible traces of air bubbles. The final accurate settings of the gages are then made by trial and error, alternately adjusting the vacuum by means of the vacuum pump and adjusting the conduit velocity by means of the main valve in the pipe line. As the vacuum increases the cavitation pockets appear and become stabilized in size, shape, and location as the final adjustment is made.

The computations for gage settings in the diverging-tube apparatus are exactly similar to the foregoing, except that a higher conduit velocity must be assumed. In a diverging tube of infinite divergence and negligible friction, the ideal throat velocity required to raise the pressure from a vapor pressure of one foot of water at the throat to an atmospheric pressure of 14.7 lb per sq in., or 33.9 ft of water, at the exit would be $\sqrt{2\,(32.2)\,(33.9 - 1.0)}$ or 46.1 ft per sec. Making some allowance for friction and exit losses requires a velocity somewhat higher than this—say about 50 ft per sec—in the model conduit, since the latter is used as the venturi throat.

In many cavitation tests it is not necessary to compute the gage setting with as much refinement as that used in the foregoing illustration. For example, in the enclosed-tank apparatus it is possible to produce so low a vacuum at the conduit reference section as to cause long cavitation streamers to open up in the body of the water, usually along the axis of the conduit and in contact with the glass window. Under such conditions, the cavitation potentialities simulated at all places along the model conduit surfaces are far more severe than

those of the prototype. If a design shows no cavitation pockets in contact with the conduit surfaces under such operation, it can be classed as excellent from the cavitation standpoint. In the diverging-tube apparatus, a similar exaggeration of the severity of the prototype cavitation conditions can be secured by increasing the velocity in the model conduit.

Behavior of Original Madden Dam Conduit Entrances in Models

Those dimensions of the original entrances of the Madden conduits which are significant from a hydraulic standpoint are shown in Figs. 1 and 11. Fig. 11 also indicates at point T the approximate location of the pitting in the prototype. As noted in Fig. 1, the conduit entrances are not symmetrical about a horizontal plane through the conduit axes, but the bottoms of the stop-log recesses terminate in a horizontal step or shelf 18 in. below the level of the conduit bottom. This shelf suppresses the contraction of the jet to such an extent that no pitting on the bottom occurs in the prototype or models, and no visible cavitation pockets appear in contact with the bottom in the models.

Fig. 10(*a*) shows a 1 : 15 scale model of one of the original Madden conduits, split along a vertical diametrical plane. Fig. 10(*b*) shows a 1 : 20 model of a pair of the same conduits split along a horizontal diametrical plane, the model being inverted so that the portion reproducing the top of the conduit is seen through the glass window. Fig. 10(*c*) shows a 1 : 40 scale transparent model of one of the original Madden conduits. Figs. 10(*a*) and 10(*b*) show the first two models being tested in the enclosed-tank apparatus and the third (Fig. 10(*c*)) in the diverging tank apparatus under pressures equivalent to a 168-ft head in the prototype. A photograph showing pitting in one conduit of the twin model of Fig. 10(*b*) is reproduced in Fig. 12. In all these views the large cavitation pocket against the top of the conduit just inside the entrance is very conspicuous. No other cavitation pockets are present anywhere in the entrance region, except the faint streamers just visible near the conduit bottom in Fig. 10(*a*). These streamers do not come in contact with any solid material, and therefore do not cause pitting. In a test of a 1 : 15 scale model of one of the Madden conduits after installation of the bellmouth, the size of the cavitation pocket was seen to be nearly as large as in the original conduits.

One of the most puzzling problems encountered during the conduct of the tests described herein was to find a reason for the fact that in the original Madden conduits, and in those models of these conduits which were actually subjected to pitting tests, deep erosion of the side-walls of the conduits occurred at places where absolutely no cavitation could be observed visually in the uneroded models. Contours of the side-wall erosion in Madden Conduit 5 after the first season's operation are shown in Fig. 3, whereas the photographs reproduced in Fig. 10 indicate the area covered by the aforementioned large cavitation pocket. The edges of this cavitation pocket are in contact with the side-walls of the conduit near the top, but, as noted in Fig. 3, the side-wall erosion not only occurs in the area touched by this pocket, but also extends a considerable distance downstream and downward toward the bottom of the conduit. The point of deepest side-wall erosion is entirely outside the area touched by the cavitation pocket in Fig. 10(*a*).

The explanation of this phenomenon is as follows: There is an outstanding difference between the process that controls the growth of the pit eroded in the top of the conduit and that which controls the growth of the pits eroded in the two side-walls. However, the excavation of all three of these pits is started by the action in the same large cavitation pocket. In the case of the pit in the top of the conduit, the enlargement and deepening of the eroded region provides a stilling pool which helps to absorb the energy of the rapidly moving water filaments and to cause the vapor cavities to collapse in water rather than in contact with solid material. Eventually, this pit tends to attain a certain

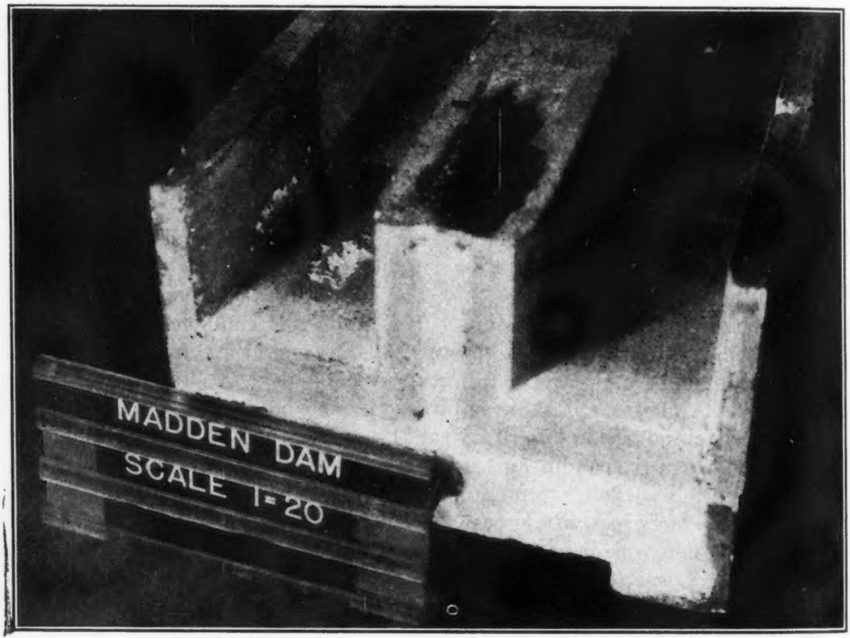


FIG. 12.—ORIGINAL CONDUIT ENTRANCE, MADDEN DAM, SHOWING THE PITTING IN A 1 : 20 SCALE MODEL AFTER ONE HOUR OF OPERATION (PROTOTYPE VELOCITY, ONE CONDUIT, 128 FT PER SEC)

stability in dimensions. The action is closely analogous to the erosion of a pit in the bed of a river below a dam whose apron is not specifically designed to prevent such erosion. The scouring out of such a pit is very rapid until it becomes large enough to stabilize the hydraulic jump and form a natural stilling pool where the energy of the spillway jet is dissipated. As this condition is attained, the growth of the excavation becomes slower and finally ceases.

In the case of the pits in the side-walls of the Madden conduits, the process of growth is radically different from the foregoing. As shown in Fig. 12, the pit first develops along the edge of the large cavitation pocket, where especially intense impacts are caused by collisions between the swiftly moving water of the streamline filaments and the broken masses of water and vapor hurled against

these filaments from the interior of the pocket. As the pit develops, it becomes in itself a source of cavitation. As the swift water of the streamline filaments passes over the edge of the pit, vapor cavities form beneath or pass in from the main cavitation pocket, and the violent collapse of these vapor cavities enlarges the excavation. This growth of the pit in the downward direction, or toward the bottom of the conduit, causes it to eat in under the swift water of the streamline filaments and thereby tap new sources of destructive energy. Along this edge of the pocket there is no opportunity for the impact to become cushioned by enlargement of the excavation, but swift water is constantly fed in over the edge of the pit, inducing violent collapse of vapor cavities at a place in immediate contact with solid material. This action causes the excavation to grow rapidly in a direction toward the bottom of the conduit and downstream, and thus the pit eventually extends itself into a region far beyond the boundary of the original cavitation pocket.

The value of model studies of cavitation by visual observation of the occurrence of cavitation pockets is not destroyed by the fact that such observation does not furnish full information regarding the final location of the pitting. If any cavitation pockets are observed in contact with the surfaces of a model conduit, it is safe to assume that destructive effects will be induced in the prototype, although the exact nature of such effects may be uncertain.

Experiments to Find a Material to Simulate Pitting

In spite of theoretical difficulties, it was thought that there might be a possibility of developing, by empirical methods, some material that would pit at a rate determinable experimentally in terms of the intensity of cavitation. It would have been very convenient to make a model conduit of such a material, run it in the cavitation apparatus for a definite number of hours, remove it and measure the depth of pitting, if any, and then state that the same degree of pitting must occur at the same location in the prototype after a definite period of operation. A considerable amount of thought and work was expended in efforts to develop such a material, but these efforts were not rewarded with complete success.

The accurate measurements available of the depth and location of destructive pitting in the original conduit entrances of the Madden Dam furnished an excellent opportunity to check the pitting in a model against that of a prototype. Materials were found which (after a reasonable number of hours of operation in models of the original Madden conduits) would undergo pitting simulating that of the prototype quite accurately in depth and location. However, full success was not obtained in controlling the uniformity of manufacture of these materials to such a degree that they would always pit to the same depth in the same number of hours.

Portland cement concrete or mortar, of approximately the same strength as that of the prototype, is altogether too hard for use in pitting tests in the enclosed-tank apparatus. In a 1 : 15 scale model of a Madden conduit constructed of this material, the depth of pitting after more than a hundred hours of operation amounted to less than an eighth of an inch, whereas the desired depth of cutting to simulate the prototype pitting was more than an inch.

In order to obtain more rapid rates of pitting, various materials were tried in the model conduits, the best results being obtained with mixtures of plaster of Paris, portland cement, and sand. When a mixture of these materials was made sufficiently weak to pit to the required depth in two or three hours of operation, there was a general tendency for the surface to soften or dissolve and wash away. This action was more rapid in places where the velocity was high, and cutting from this cause could not be distinguished easily from that due to cavitation. This difficulty was overcome by painting the surface of the material with a grout of portland cement and water. After setting twenty-four hours or more this gave a brittle skin surface which afforded perfect protection against erosion by smoothly flowing water, but was easily disintegrated by the hammering effect of cavitation.

The following material when used in a 1 : 15 scale model of the original Madden conduits was found to reproduce the pitting approximately to scale during a test period of about two to four hours: 1 part portland cement, 12 parts plaster of Paris, and 16 parts sand, mixed with 7.23 cc of water per ounce of dry material. The wash used on the surface consisted of a portland cement grout containing 17 cc of water per ounce of cement. The pitting obtained by operating a model made of this material for one hour is shown in Fig. 12.

Although skin formed by the portland cement wash was quite successful in eliminating general erosion and localizing the pitting to the cavitation areas, much difficulty was found in duplicating the thickness and total strength of this skin in models built at different times. In fact, this was one of the principal difficulties that prevented the attainment of full success in securing sufficient uniformity in the manufacture of model conduits to justify the placing of complete reliance on the results of pitting tests on such models.

Model Studies of Various Methods Proposed to Eliminate or Minimize Cavitation Damage in the Conduit Entrances of Madden Dam

In the course of the model studies leading toward a solution of the cavitation problem at Madden Dam, tests were made of various methods proposed to eliminate or minimize damage in the conduit entrances. These various methods or measures may logically be arranged in two general groups—namely: (1) Those which have as their purpose the provision of conduit boundaries conforming more nearly to the paths naturally sought by the boundary filaments of the flowing water, and (2) those which have as their purpose a general adjustment of pressure within the entire conduit or upstream part of it. Among the methods tested in the laboratory may be mentioned the following:

Group 1.—

(*a*) Reconstruction of the conduit entrance to provide more liberal curvature;

(*b*) Extension of the conduit entrance upstream from the face of the dam to provide a bellmouth entrance (with and without stop-log slot filler or cover); and

(*c*) Construction of a venturi throat in the critical region of the conduit.

Group 2.—

(*a*) Operation of the conduits at partial gate closure;
(*b*) Provision of a constriction or nozzle at the outlet end of the conduit;
(*c*) Venting of the conduit by admission of air at the stop-log slot; and
(*d*) Venting of the conduit by admission of air at the slide gates.

In addition to studies of these various methods proposed for solving the problem, tests also were made to indicate what effect debris accumulations on the trash racks would have in altering flow conditions at the entrances to the conduits, and what effect such altered flow conditions would have on cavitation within the entrances.

In the case of the Madden Dam, all of the measures of Group 2 were found to have the disadvantage of reducing the discharge capacity of the conduits, and were therefore considered applicable only as temporary expedients. In Group 1, under item (*a*), extensive tests on streamlining the conduit entrances resulted in the development of "Design No. 2," shown in Fig. 13. This was

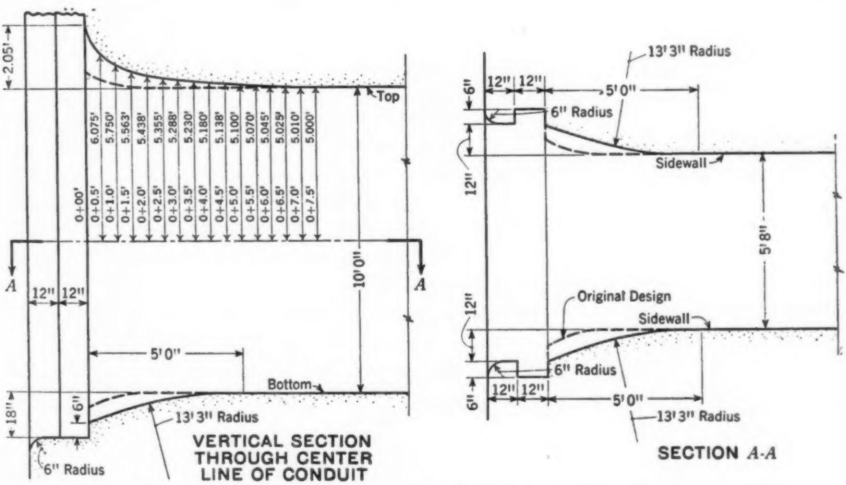


FIG. 13.—DESIGN NO. 2, MADDEN DAM CONDUIT ENTRANCE, ADOPTED FOR CONDUITS 1 TO 4

recommended for actual incorporation in all the units except Conduits 5 and 6. The model studies showed this design to be free from cavitation even under a head much greater than the maximum possible in the prototype. Under item (*b*) it was found that in Conduits 5 and 6, which were already provided with bellmouths extending upstream from the entrance, cavitation could be eliminated or minimized by providing the stop-log slot with a strong cover or filler. Under item (*c*), the venturi throat was found to have no advantage over "Design No. 2" to compensate for its greater cost. In the case of the Tygart River Dam, the models showed only faint or incipient flashes of cavitation at the conduit entrances. However, since the conduits were to be steel-lined and the entrance castings already had been fabricated and delivered, it was decided

to complete the installation in accordance with the original design, but to observe carefully the effect of operation by regular inspection of the conduits, and to make such repairs as necessary if pitting of the entrance casting and lining is found to occur. The entrance bells of the Hiwassee, Bluestone, and Redbank conduits, designed with exceptionally liberal curvature in the light of information regarding the Madden difficulties and the model studies, proved to be entirely free from cavitation, but in the cases of the latter two of these three dams the models detected quite serious cavitation at gate slots in the preliminary designs, which was later eliminated through revision of these designs.

Acknowledgments

Various engineers from the Panama Canal Office, the Pittsburgh and Huntington (W. Va.) District Offices of the United States Engineer Department, and the Hydraulic Research Laboratory of Carnegie Institute of Technology have made contributions toward solving the problems of controlling or eliminating cavitation in the structures which have been mentioned. W. E. R. Covell, M. Am. Soc. C. E., district engineer at Pittsburgh, has taken a keen and helpful interest in the cavitation studies. E. S. Randolph, M. Am. Soc. C. E., chief of the Design Division for the Panama Canal, had a responsible part in the design, construction, test, and repair of the conduits of the Madden Dam, and contributed extensively to the original investigations leading to the tests described herein. J. C. French, junior engineer in the Canal Zone, assisted in the model studies of the Madden conduits. W. S. Hamilton, Jun. Am. Soc. C. E., instructor in Civil Engineering at Carnegie Institute of Technology, and Harold A. Thomas, Jr., instructor in Sanitary Engineering at Harvard University, in Cambridge, Mass., worked extensively on the cavitation experiments. C. F. Merriam, hydraulic engineer for the Pennsylvania Water and Power Company, reviewed the manuscript of this paper and contributed a number of valuable suggestions. For authority to publish information pertaining to the Madden Dam the writers are indebted to Brig. Gen. C. S. Ridley, Governor of the Panama Canal Zone.

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

DISCUSSIONS

TREND IN HYDRAULIC TURBINE PRACTICE
A SYMPOSIUM

Discussion

BY MESSRS. I. A. WINTER, AND L. M. DAVIS

I. A. WINTER,[36] ASSOC. M. AM. SOC. C. E. (by letter).[36a]—Mr. Mattern has suggested that a brief description be given of the power step-up method which the writer has proposed for predicting the performance of prototype turbine installations from test results on strictly homologous models. The method involves a comparison of power output between model and prototype and in this respect offers a comprehensive means for predicting field efficiency. Basically, the method requires an increase in efficiency for an increase of horsepower of prototype over model and a decrease in efficiency for a decrease in horsepower.

The proposed method further requires that a facsimile velocity-vector diagram be obtained for both model and prototype at the inflow and outflow of the runner when the final discharge is determined for the field test. Thus, a change in discharge between model and prototype for a given gate opening requires a change in phi (ϕ_m) for each point considered on the horsepower-phi and efficiency-phi curves obtained in the laboratory. One or two approximations are usually sufficient to obtain the correct phi on the horsepower and efficiency curves for use in comparison with the horsepower of the unit as determined in the field.

Another factor influencing the relative power of model and prototype is the inflow and outflow area and inlet diameter of the turbine runner. It is readily seen that a change in orifice area through the turbine runner with respect to the model would produce a change in horsepower without necessarily changing the turbine efficiency. This very important factor is evaluated by accurately measuring the outflow area of the model and repeating these measurements on each runner of its prototype. The necessary correction factor may than be

NOTE.—This Symposium was published in November, 1939, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: January, 1940, by Messrs. W. S. Pardoe, and Donald H. Mattern; March, 1940, by Messrs. Lewis F. Moody, and R. E. B. Sharp; April, 1940, by Messrs. Martin A. Mason, and E. Shaw Cole; May, 1940, by Messrs. Paul L. Heslop, and J. D. Scoville; and September, 1940, by K. W. Beattie, Assoc. M. Am. Soc. C. E.

[36] Senior Engr., U. S. Bureau of Reclamation, Denver, Colo.

[36a] Received by the Secretary September 5, 1940.

applied to account for lack of similarity between model and prototype. These data should be obtainable from routine records of the turbine manufacturer.

The following example of computation will serve to illustrate the steps necessary in determining the field efficiency and has been suggested by Mr. Mattern in connection with an index test made on units 1 and 2 at the Wheeler power plant of the Tennessee Valley Authority:

The test was conducted at 50-ft head, or $\phi_m = 1.74$. Since the gate orifices are homologous, the power output of the prototype should theoretically step up directly by the square of the diameters and the three-halves power of the head. Reduction of losses, however, is such that the power of the prototype increases, and the actual phi at which the runner is operating must have changed. The step-up method begins by first taking this into account in Equation (1); thus:

$$\phi = \frac{p_m}{P}\,\phi_m \qquad (20)$$

in which: ϕ_m = phi, or the ratio of peripheral velocity of runner to spouting velocity of water at the head used for the field test; ϕ = effective phi at which runner is operating; p_m = field power stepped up directly from model performance at ϕ_m; and P = prototype power. For $\phi_m = 1.74$, Equation (20) becomes $\phi = \frac{47{,}000}{48{,}000} \times 1.74 = 1.704$ (first approximation).

Going to the model test data and picking off the values for the proper gate opening, it is found that the field power p_ϕ, stepped up from the model performance at ϕ, = 46,650, and the model efficiency (e) at ϕ = 87.2%. The discharge can then be computed:

$$q = \frac{550\ p_\phi}{62.5\ h\ e} \qquad (21)$$

in which: q = discharge computed from model efficiency and stepped up power at ϕ; and h = head required to produce stepped up model power at 100% model efficiency (e), using p_ϕ and e. From Equation (21), $q = \frac{550 \times 46{,}650}{62.5 \times 50 \times 0.872}$ = 9,416 cu ft per sec.

The theoretical head required to produce the foregoing power is then determined. By this means it is possible to work directly with the fundamental quantities without adjusting for variable losses; thus:

$$h = \frac{550\ p_\phi}{62.5\ q} \qquad (22)$$

From Equation (22), $h = \frac{46{,}650 \times 550}{9{,}430 \times 62.5} = 43.60$ ft.

Outputs are proportional to the three-halves power of the heads so that $\frac{p_\phi}{h^{1.5}} = \frac{P}{H^{1.5}}$; from which,

$$H = \frac{h\ P^{0.67}}{p_\phi^{\ 0.67}} \qquad (23)$$

From Equation (23), $H = \frac{43.6 \times (48{,}000)^{0.67}}{46{,}650^{0.67}} = 44.44$.

Discharge is proportional to the square root of the heads so that

$$Q = \frac{q \sqrt{H}}{\sqrt{h}} \qquad (24)$$

in which Q = discharge of prototype. From Equation (24), $Q = 9{,}416 \times \frac{\sqrt{44.44}}{\sqrt{43.60}} = 9{,}506$ cu ft per sec. The final value desired is the prototype efficiency E, and this can be computed from the fundamental formula

$$E = \frac{550\,P}{62.5\,H_T\,Q} \qquad (25)$$

From Equation (25), $E = \frac{48{,}000 \times 550}{50 \times 9{,}506 \times 62.5} = 88.8\%$ (based on first approximation). The corresponding result from the Moody formula would have been

$$E = 100 - (100 - e)\left(\frac{d_m}{D}\right)^{0.25}\left(\frac{h_m}{H_T}\right)^{0.01} \qquad (26)$$

From Equation (26), $E = 100 - (100 - 87.2)\left(\frac{16}{264}\right)^{0.25}\left(\frac{3.8}{50}\right)^{0.01} = 93.81\%$.

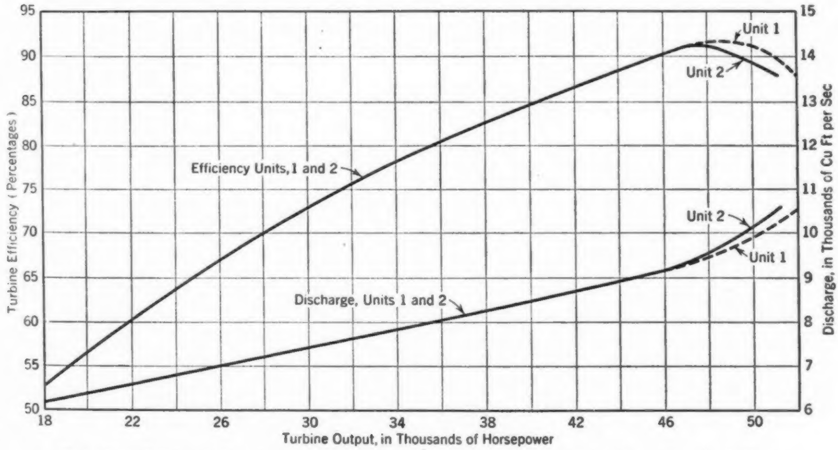


FIG. 31.—TURBINE EFFICIENCY CURVES—UNITS 1 AND 2 AT 49 FT NET TURBINE HEAD

The preceding example indicates the greatly different results obtained by the power step-up method and the formula proposed by Professor Moody. The performance curve shown in Fig. 31 for units 1 and 2 of the Wheeler power plant has been derived by the power step-up method based on comprehensive index tests fully equal to an acceptance test, in regard to the number of observations made and in degree of accuracy of measurement and computation. These curves may be considered as very satisfactory.

Where a hydraulic laboratory has established a definite efficiency-increase ratio between laboratory and field tests, based on years of experience, no necessity is seen for the use of a step-up formula, since in the final analysis the laboratory differential must be fulfilled if there is sufficient similarity between model and prototype. It is not conceivable that any manufacturer would fail to meet guaranteed efficiency where final acceptance of the turbine is made on the basis of tests on laboratory models.

However, results of field tests do not confirm full attainment in all cases of the laboratory and field differential, and hence the power step-up method is proposed, since it definitely places responsibility for fulfillment of field requirements without the necessity of a greatly involved testing procedure, results of which might be questionable due to unfavorable flow measuring conditions.

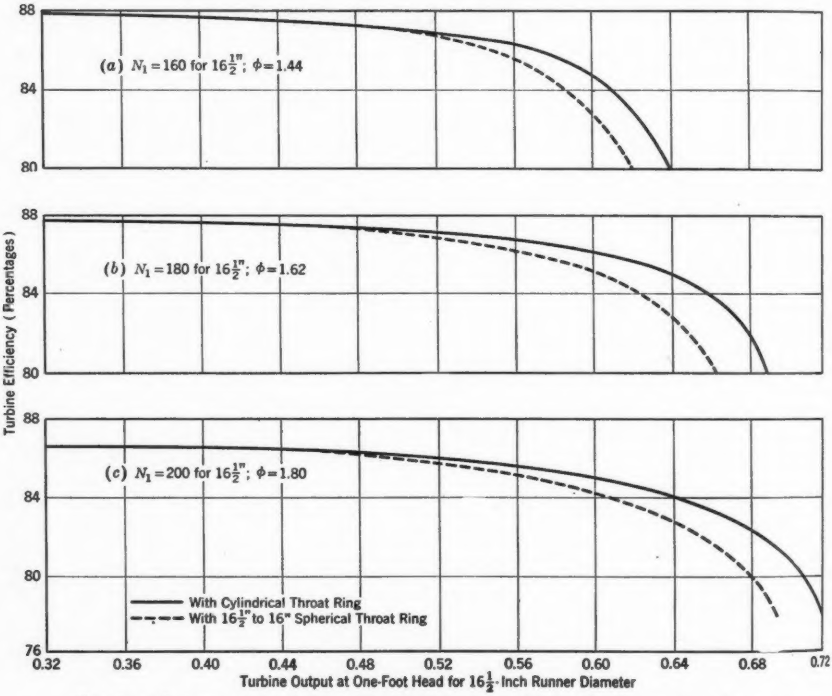


FIG. 32.—COMPARISON OF MODEL TEST RESULTS ON 16½-IN. DIAMETER, FIVE-BLADE ADJUSTABLE RUNNER

In the first paragraph of his discussion, and in Fig 20, Mr. Sharp presents data comparing the performance of a runner model, 11 in. in diameter, tested in a spherical shaped throat ring, with its performance tested in a cylindrical throat ring. These data are at variance with the test results on which the writer based his statements in the last paragraph under "Turbines."

The latter test results were made on a five-blade adjustable runner with a nominal diameter D of 16.5 in., under about 12 ft of head, in the hydraulic

laboratory at Newport News, Va. In Fig. 32 comparisons of enveloping efficiency against unit power are made between a cylindrical and spherical throat ring at unit speeds of 160, 180, and 200. The corresponding values of peripheral coefficient (ϕ) are 1.44, 1.62, and 1.80 and are about the same as those noted in Fig. 20. The test results indicate a definite advantage of the cylindrical throat over that particular spherical throat ring. The smallest diameter of the spherical throat ring was 16.00 in., or 0.970 D, as compared with Mr. Sharp's value of 0.983 D. Since the burden on the draft tube varies inversely as the fourth power of the throat diameter, the 16-in. spherical throat places a burden on the draft tube that is about 12.5% greater than that of the $16\frac{1}{2}$-in. cylindrical throat. Consequently, it should not be surprising that the cylindrical throat shows better performance at the higher discharges. It is true that the greater clearance at the higher blade angles causes some greater leakage loss. However, that may be overbalanced by the eddy loss created downstream from the somewhat abrupt curvature of the spherical shape.

The test results at Newport News, in general, indicate that with the adjustable runners of higher specific speed there is an advantage of the cylindrical over the spherical throat, whereas in the case of the lower specific speeds there is not much difference. For the lower specific speeds a shape has been adopted, that is intermediate between the cylindrical and spherical shapes, in which the sectional radius of curvature is made about 1.6 versus 0.50 D. The throat is made 0.98 D and is located well downstream from the blade axis at a distance 0.175 D versus Mr. Sharp's 0.125 D. This compromise combination places the throat nearer to the elevation of the blade tips when fully open and reduces the change in flow angle when passing from the throat to the draft tube.

It may be of interest to observe that a cylindrical throat ring design was adopted in 1939 for the high-speed units in the Vargon plant in Sweden.[37] These are the largest units of that type in operation.

L. M. DAVIS,[38] Assoc. M. Am. Soc. C. E. (by letter).[38a]—In his theoretical analyses of how he arrived at the fifth root exponent for the Moody step-up formula, Professor Pardoe has made a valuable contribution to the subject. As is well realized by all parties, however, the correct exponent must be determined from experience; and thus more comparisons are necessary between model and prototype for the large sizes.

Mr. Mattern has emphasized a number of important points in his discussion. Fig. 18 is of particular interest since it shows the variations in expected efficiency which result from applying the Moody step-up formula in the three ways mentioned. As Mr. Mattern states, Professor Moody recommends using the peak efficiency of the model, regardless of speed or blade angle, in computing the increment of efficiency from model to prototype, and then adding this increment to all model efficiencies to obtain the expected efficiency. This method is now in general use. In his oral discussion of this paper (at the meeting of

[37] *Power*, December, 1939, pp. 744–746.

[38] Hydr. Test Engr., Pennsylvania Water & Power Co., Holtwood, Pa.

[38a] Received by the Secretary September 20, 1940.

the Power Division, Rochester, N. Y., October, 1938), Professor Pardoe objected to this method since at zero power it would result in an efficiency equal to the increment, which, of course, is impossible. He suggested that a more rational method of applying the increment would be to multiply it by the ratio,

$$\frac{\text{Discharge at best efficiency for head and speed conditions in question}}{\text{Discharge corresponding to the efficiency point to be stepped up}} .$$

The result would then be added to the model efficiency. The tendency would then be to give lower expected efficiencies at all points below the point of best efficiency, and higher for all points above, than the usual method. This would be a step in the right direction because ordinarily the prototype efficiency has such a tendency in comparison to the model efficiency.

Professor Moody has shown that the exponent used in the Moody step-up formula was determined from comparative model and prototype tests available at that time. This will answer Mr. Mason's question of the advisability of using a theoretical formula which has a none too stable background.

The remarks regarding the limitations of the pitot tube to which Mr. Cole took exeption were, to say the least, vague. However, the idea in mind was that in a large unit intake having two or three intake passages of massive concrete it would be impracticable to attempt to measure water with a pitot tube, due to the mechanical difficulties and the expense involved.

Mr. Scoville states that on the 5,000-hp service unit at Bonneville the field test gave efficiencies 1% above the expected at higher loads than the best efficiency and 1% lower than the expected at lower loads. This adds weight to Professor Pardoe's suggestion for applying the step-up.

Mr. Beattie has presented some very interesting information based on experience. As he states, the behavior of the power and discharge characteristics of different turbine models is quite different near the point where cavitation starts. No doubt the location in the turbine where cavitation takes place dictates the behavior of the power and discharge characteristics. It seems quite logical that, if cavitation occurred well upstream from the point of smallest discharge area in the turbine, the discharge might be reduced, due to the turbulence set up by cavitation. If this reasoning is accepted as a cause for reduced discharge with the start of cavitation, it might be further reasoned that by a slight alteration in blade shape this low pressure zone occurring upstream from the point of maximum velocity might be eliminated and the cavitation characteristics thus improved.

The basis for the statement that failure of the unit power and unit discharge breaks to coincide is an indication of faulty design is that, in a turbine of perfect design, cavitation may be expected to occur first at the point of maximum velocity, which results in minimum pressure. This will naturally be at the minimum discharge area in the turbine where it seems logical to expect an effect on both power and discharge characteristics. However, if cavitation starts at one or more points on the blades away from this control point, the power may be effected without any change in discharge or the discharge may even be decreased as mentioned herein.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

TRANSIENT FLOOD PEAKS

Discussion

By Henry B. Lynch, M. Am. Soc. C. E.

Henry B. Lynch,[53] M. Am. Soc. C. E. (by letter).[53a]—The purpose of this paper was to call attention to, and present data relating to, the excessive cross sections characteristic of the so-called "cloudburst" type of floods, as well as to give a satisfactory explanation of them. Such floods have not received the consideration due them from engineers. This condition is due equally to lack of data and lack of discussion. The areas generating the floods have almost always been uninhabited and devoid of rain gages. As a result, it has been simple to assume that floods were caused by extremely heavy rates of mountain rainfall.

The Southern California flood of January 1, 1934, was unique among floods of this type because of the mass of available, pertinent data. In the Sierra Madre, within the area generating the floods, were five rain gages, two of which were automatic. Eight other gages were located in the thickly populated section within one mile of this area. These gages gave consistent rainfall data. Many people living within, or immediately adjacent to, the mountains were interviewed. All evidence showed that rainfall rates were only moderately high. The excessive cross sections were measured on all sides of the gages and inhabited locations. As a result of the information available, the usual explanation of a "cloudburst" was not advanced by any one who studied the phenomena at first hand, so far as the writer knows.

This paper has evoked much discussion, both favorable and unfavorable. The writer wishes to acknowledge a debt to all who discussed it, whether in agreement or in objection. The discussion has furnished important data, and thrown much light on various matters connected with these floods.

There is no controversy as to the occurrence of large cross sections or high velocities. Those familiar with the ground, who discussed the paper, found

Note.—This paper by Henry B. Lynch, M. Am. Soc. C. E., was published in November, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: January, 1940, by Ivan E. Houk, M. Am. Soc. C. E.; March, 1940, by Messrs. Gordon R. Williams, and Donald M. Baker; April, 1940, by Messrs. James M. Fox, F. C. Finkle, A. L. Sonderegger, and Harold C. Troxell and R. Stanley Lord; May, 1940, by Messrs. Karl J. Bermel, and R. W. Davenport; June, 1940, by Walter J. Wood and Maxwell F. Burke, Assoc. Members, Am. Soc. C. E.; and October, 1940, by Franklin Thomas, M. Am. Soc. C. E.

[53] Cons. Engr., Los Angeles, Calif.

[53a] Received by the Secretary October 17, 1940.

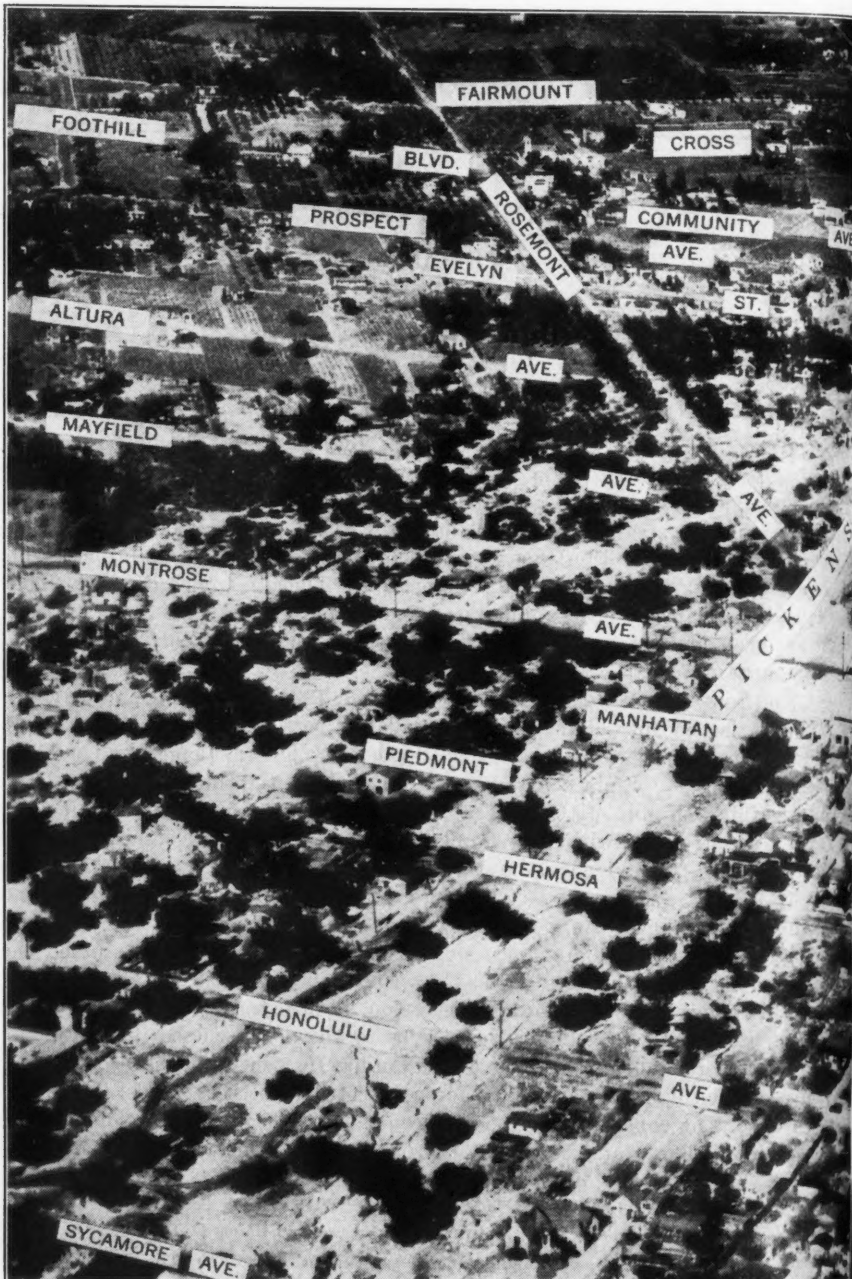

FIG. 22.—PATH OF DESTRUCTION AND

Overflow from Pickens Canyon

similar facts. There is definite disagreement as to causes, however, and as to runoff conditions during the flood. Objections to the explanation given by the writer have practically all revolved around the proposition that rate of runoff never exceeds rate of rainfall, except in certain well known and simply explained cases, such as the failure of dams.

The writer has no doubt that all of the factors enumerated in the objections were present during the flood, to some extent, or at some time. It is not believed, however, that any of them, or all of them combined, were sufficient to account for the destruction wrought by the flood, or for the other facts disclosed.

The discussion has revealed three basically differing explanations to account for the observed phenomena:

(1) That the large cross sections resulted from a retardation of the stream flow as it passed down the channels, and its consequent accumulation, this retardation being caused by a mass of water and debris, which acted somewhat as a moving barrier, traveling downstream at a slower rate than the velocity of the water behind it, and gradually impounding such water;

(2) That these flows were slow-moving bodies of water, very heavily loaded with debris and advancing somewhat as a mud flow, and that the rate of flow of water in the streams was never greater than would be accounted for by the rate of rainfall; and

(3) That the cross-sectional areas of channels were temporarily greatly reduced by accumulations of debris. Over a part, or all, of the widths of the debris, streams of water flowed at rates of discharge that could be explained at all times by the rates of rainfall on the drainage area. These streams might attain high surface velocity. Subsequent cutting removed most of the temporary channel loads. As a result of these conditions, cross sections measured subsequent to the flood did not correctly represent the situation that existed during the flood.

The first explanation is that adopted by the writer. Waves were constantly augmented by high-velocity surface water from upstream which advanced to the wave front and then largely dissipated its energy in turbulent vertical action upon the stream beds.

Although the data do not permit an estimate of the instantaneous flow, perhaps a rough approximation to the lower limit of flows directly behind wave fronts is feasible. This involves the surface velocities as computed from splashes.

In Hall-Beckley Canyon, surface velocity has been computed as not less than 25 ft per sec near the left bank of the stream. If clear water, only, had flowed in this canyon, the properties of the stream bed would require a hydraulic radius of 2.93 ft in order for the surface water to reach this velocity. At this point in the canyon, a section of 265 sq ft is reached before the hydraulic radius becomes 2.93 ft. For such a cross section the flow of clear water needed to sustain this surface velocity is 5,300 cu ft per sec, equal to a runoff over the drainage area at the rate of 11 in. of depth per hour. When allowance is made for the heavy debris load, then the quantity of water flowing is increased by the quantity necessary to impart velocity to the debris.

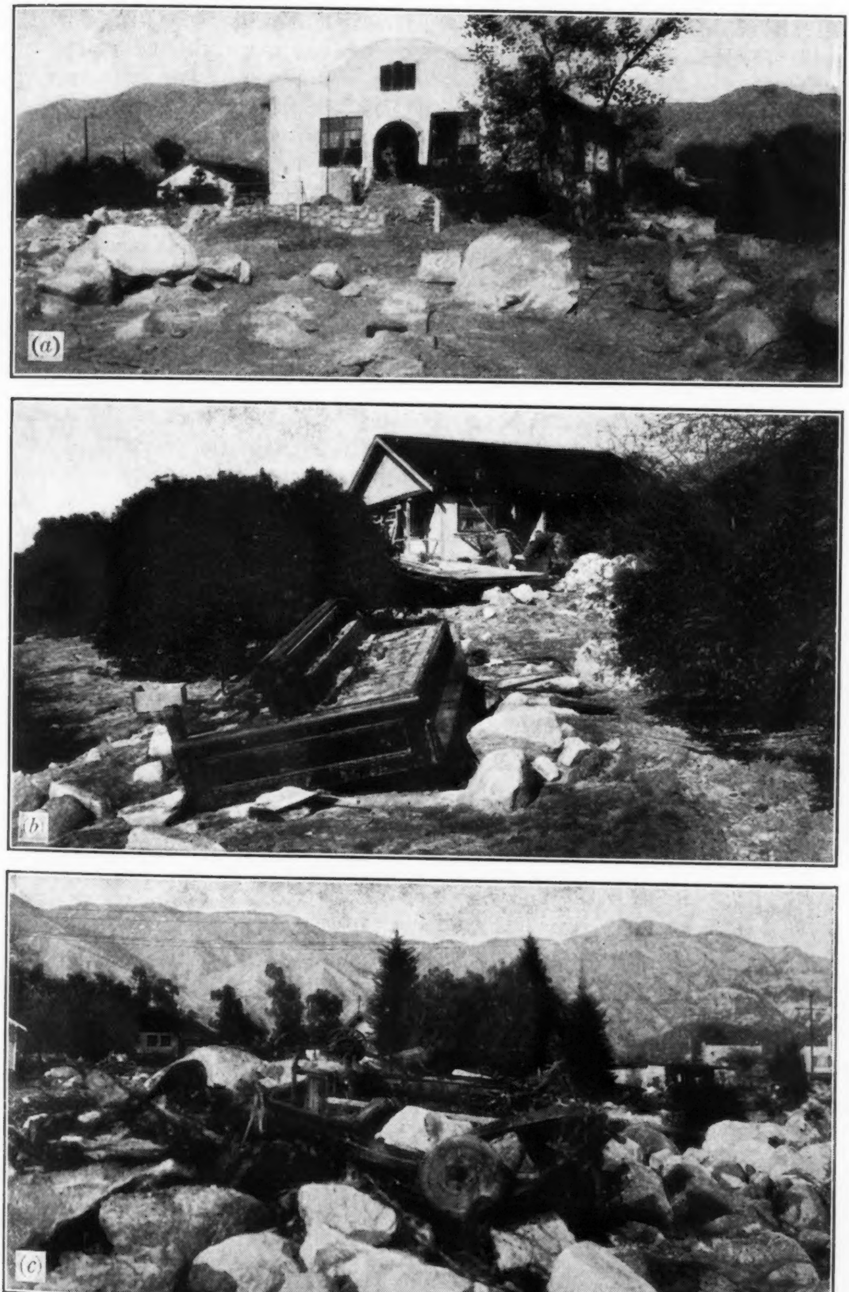

FIG. 23.—BOULDERS AND DESTRUCTION LEFT BY OVERFLOW OF PICKENS CHANNEL

The second explanation proceeds upon assumptions of average velocity and ratio of volume of water to debris which have been given variously by different writers. The following are about as low as were given by any one: Average velocity = 10 ft per sec, and ratio of water = one third of volume.

On this basis the rate of flow of water and debris above tops of check dams in Hall-Beckley Canyon becomes 6,800 cu ft per sec, and clear water is 2,266 cu ft per sec. This is at the rate of almost 5 in. in depth per hour over the drainage area.

The measurements at this point did not seem to be open to question. It does not appear to the writer to be possible to invoke the second explanation in Hall-Beckley Canyon, even when average rate of flow and percentage of clear water are used, which are lower than the most extreme values that any one has suggested. This same statement applies to Winery, Mullaly Fork, West Fork of Eagle, Shields, Dunsmuir, Cooks, and Blanchard canyons.

The third explanation calls for positive physical elevation of the stream beds from 6 to 20 ft throughout almost the entire length of the channels. These fills are stated to be formed during the earlier stages of the flood, when waters are rising, and to be composed of boulders, gravel, sand, and fines. The fills are scoured out at a later stage of the same flood, leaving, however, numerous evidences of their former presence. The writer saw no places or photographs to indicate that this had occurred except in a few fills in protected locations within the channel. These were never more than 3 ft deep, and were not typical of usual conditions. Fig. 15 was from a photograph taken after an ensuing small flood in October, 1934,[54] almost a year later. It does not relate to the type of action described, but to one almost diametrically opposite. This fill was composed of almost pure sand, about 5 ft deep. It was made at a late stage of the January flood, when velocities were so low that the flow carried only sand, and deposited a large part of that. Afterward it was scoured out by the rising water from a subsequent storm. The same thing occurred at that time in many places in the lower channels. It can be duplicated whenever a small flood follows a major one. The early stages of a large flood scour out all evidence of these deposits of loose sand.

On the inhabited slopes below the mountains many of the difficulties of the second and third explanations are common to both. Fig. 22 shows the cone below Pickens Canyon and the widespread destruction from this flood. The catchment area is 1.81 sq miles. Any satisfactory explanation must account for conditions here as well as in the canyons. Fig. 23 shows boulders thrown outside of Pickens channel downstream from the mountains, in the flood of January 1, 1934. Figs. 23(*a*) and 23(*b*) are views on the east side and Fig. 23(*c*) is a view on the west side. The boulder in the foreground of Fig. 23(*a*) weighed about 13 tons. If either the second or the third explanation is correct, such boulders were deposited, together with gravel, sand, and fines, by comparatively small streams of water. Later in the course of the flood, underloaded streams of water, also comparatively small, are supposed to have overflowed from the channel and washed all of the gravel, sand, and fines from each of these areas.

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[54] "Flood in La Cañada Valley, California, January 1, 1934," by Harold C. Troxell and John Q. Peterson, U. S. Geological Survey *Water Supply Paper No. 796–C*, 1937, Plate 24–A.

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

NORRIS DAM CONSTRUCTION CABLEWAYS

Discussion

BY ADOLPH J. ACKERMAN, M. AM. SOC. C. E.

ADOLPH J. ACKERMAN,[10] M. AM. SOC. C. E. (by letter).[10a]—The authors of this paper have rendered a valuable service in describing the design, construction, and operation of modern cableways. It is seldom that information on heavy-duty construction equipment is available in such detail, and the data with respect to design, performance, and operation, including first cost and operating costs, are very valuable. The cableways used in the construction of the Norris Dam are in themselves engineering projects of sufficient importance to merit a place in the engineering literature.

The ordinary impression regarding such heavy-duty cableways is that they represent an excessive investment and as such are a questionable type of construction plant. Although this may be true for the ordinary type of construction, it is the writer's opinion that, where topography is favorable and where the volume of concrete to be handled is of substantial proportions, there is no cheaper way to build a dam than by cableway. Furthermore, it is important to recognize that cableways of small capacities and moderate cost have been used for a great variety of work in Europe, even for the construction of ships, and it is unusual that they have not found wider acceptance in the United States. Probably one of the reasons for this is that many engineers and constructors are not "cableway minded," and tend to visualize a cableway as a highly flexible and unstable unit that is not as dependable as a crane running on the firm ground. Nevertheless, those who have had experience with cableways, and have organizations of riggers and other personnel skilled in the handling of cableways, realize that the full possibilities in cableways have not been reached and that they deserve a much wider application.

The first important installation of a heavy-duty cableway, capable of handling 25 tons, was used in the construction of the Owyhee Dam in Oregon in 1929 and 1930, and in many respects this was the forerunner of a number of

NOTE.—This paper by R. T. Colburn, M. Am. Soc. C. E., and L. A. Schmidt, Jr., Assoc. M. Am. Soc. C. E., was published in December, 1939, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1940, by J. S. Foster, Esq.; April, 1940, by Gordon H. Bannerman, M. Am. Soc. C. E.; and June, 1940, by Messrs. Walter F. Weber and Blair Birdsall, and G. E. Cate.

[10] Director of Eng., Dravo Corp., Neville Island, Pittsburgh, Pa.

[10a] Received by the Secretary October 3, 1940.

heavy-duty cableways built since then. Table 10 contains a list of the important cableway installations used in the construction of dams during the ten years 1930–1940.

TABLE 10.—HEAVY-DUTY CABLEWAY INSTALLATIONS

Dam projects	Number of cableways	Capacity, in tons	Span, in ft	Track cable diameter, in in.	Hoist power, hp	HEAD TOWER		TAIL TOWER	
						Height, in ft	Type*	Height, in ft	Type*
Owyhee	1	25	1,306	3	400	65	T	45	F
Madden	1	25	1,325	3	400	100	T	100	T
Hoover	2	25	2,575	3	500	90	T	90	T
	2	25	1,405	3	500	75	T	42	T
	1	25	1,365	3	500	98	F	16	T
Morris (Pine Canyon)	2	15	960	2¼	300	100	F	35	T
Norris	2	18	1,925	3	500	75	T	110	T
Bonneville lock and power house	2	15	1,390	3	400	145	F	75	T
Bonneville spillway	2	20	2,020	3	500	90	T	223	F
Parker	2	25	1,500	3	500	75	T	42	F
Conchas	2	15	1,650	2½	250	145	T	175	F
Hiwassee	1	18	1,575	3	500	75	T	110	T
Marshall Ford	1	25	2,100	3	500	75	T	177	F
Shasta	6	25	800 to 2,600	3	500	460	F	75 to 125	T

* T = traveling and F = fixed.

Mention should also be made of the 150-ton cableway that has been installed permanently at Hoover Dam for handling heavy equipment from the rim of the canyon down to the power-house levels on either side of the river. This is an outstanding engineering accomplishment in this field and indicates the possibilities in the use of cableways.

Concrete Buckets.—At the time the Norris Dam cableways were proposed there was considerable discussion on the question of whether the concrete should be handled in buckets of 4-yd capacity or 8-yd capacity. In several earlier cases the 8-yd buckets had developed objections because the type of bucket used discharged the concrete too suddenly and with sufficient impact to disturb the concrete below which was in the process of setting. Several installations of cableways with 4-yd buckets had proved satisfactory, but it was decided finally to compromise and use two cableways with 6-yd buckets. The use of two cableways provided an economical arrangement and increased flexibility; but, as has been demonstrated on other jobs, a single cableway with an 8-yd bucket of improved design can be used successfully in the construction of a dam as large as Norris Dam.

Since the concrete bucket on a cableway is the "business end," this part of the equipment deserves special attention. At Norris Dam efforts were made to improve the method of discharging large volumes of concrete with a minimum of segregation and impact and this led to a very useful development, originated by Ross White, M. Am. Soc. C. E.—namely, an air ram built into the bucket in such a way that, when an operator in the form applied a short air nozzle to an air socket on the bucket, the ram would do the heavy part of the work in operating the gate on the bucket. By proper adjustment of the

gate the flow was controlled so that a very satisfactory discharge was developed. The bucket was so designed that it would reclose automatically while it was being hoisted, without returning to the form for latching.

Runways for Towers.—In planning the installation of a cableway, the principal problem is to find suitable sites for the tower runways. It is normally desirable to keep the span as short as possible; and under ideal conditions a flat area on each side of the canyon provides the most favorable runway sites.

However, in many cases the profile at the dam site continues to rise to a considerable height above the top of the abutment. It then becomes necessary to excavate a bench in the hillside. This means that the ground directly in front of the runway, sloping downward into the valley, introduces problems of instability with respect to supporting the horizontal forces from the tension in the track cable and operating rope. Where the bench can be excavated into solid rock, this is a less serious problem, but frequently a solid formation is not available. It may also be necessary to extend the runways by means of earth fills that have similar characteristics of instability against horizontal thrust, particularly since they are usually constructed on the downward sloping natural ground.

In the earlier types of traveling cableway towers very little consideration was given to stresses in the runways due to horizontal thrust; usually the tower was supported on ordinary railroad trucks, and the horizontal thrust was transmitted to the rails through the wheel flanges. This introduced very heavy bending moments in the wheels and frequently resulted in breakage of flanges or wheels.

An improvement over this design consisted of inclining the front wheels in the same general direction as the front part of the tower so that the wheels carried both a vertical component and a horizontal component or a resultant inclined force corresponding more or less to the inclination of the wheels. This arrangement, however, meant that the runways had to be built either of steel, on solid rock, or with heavy ties connecting the front and rear tracks to prevent spreading under the constant shifting of the wheel loads between front and rear tracks. For example, with no load on the cableway the maximum wheel load is on the rear tracks due to the counterweight. However, as heavy loads are picked up on the cableway to the point where practically all of the counterweight is lifted, the entire force—cable pull and tower weight—is balanced on the front tracks. It is interesting to note that on many European cableways the rear track is eliminated entirely and only front inclined wheels are used, the back end of the tower rocking up and down, depending on the tension in the cables.

As a further improvement in wheel arrangement and in order to make the cableway towers adaptable to a greater variety of runway conditions, the writer conceived the idea of a tower in which all horizontal thrust is taken at the rear of the tower and transmitted into the runway at that point. Since its first application at Madden Dam in the Panama Canal Zone, a number of dams have been built using this type of cableway.

The principal advantage is that the resultant inclined thrust is not transmitted into the front edge of a runway fill where it is least stable, but is directed

into the rear side of the fill and the entire fill is utilized to resist the horizontal thrust. This is clearly indicated in Fig. 1, showing the location of the horizontal thrust wheels running on an earth-fill runway for the tail towers at Norris Dam. It is apparent that if the front wheels had been inclined to carry the tremendous resultant thrust without using the rear thrust wheels, the fill would have required considerable additional width. A further advantage of this arrangement of wheels is that under all conditions of cableway loading the principal runway reactions are carried at the rear. With no cableway load, when the counterweight is most active, the principal force, of course, is on the rear wheels. At full cableway load the vertical forces on the front wheels increase but none of the heavy horizontal component is transmitted to the front wheels; this remains at the rear of the runway. This means that with a dependable arrangement of tracks at the rear, the tower could still operate with considerable misalinement or settlement of the front track.

The result of all this is that the structural arrangement of the runways and of the tower is made as simple as possible. The importance of economy in the location and design of runways is apparent from the fact that in the case of the Norris Dam installation they represented $87,000 of the total cost; very little of this construction is salvageable.

The authors call attention to the advantage of introducing the horizontal thrust at the rear of a runway, and refer to a cave-in that occurred under the head tower. Referring to Fig. 1, the runway for the head tower is shown in detail. During the construction of Norris Dam it was discovered that a clay-filled cavern in the abutment area, which was being cleaned out prior to back filling with concrete, had a chimney outlet, and it so happened that one of the trestle runway bents was located directly over this chimney. In cleaning out the clay plug in the cavern below, a displacement occurred and the trestle bent suddenly sank several feet while supporting one of the head towers. Without the rear thrust-wheel anchorage this tower would have rolled down the hillside.

Radial Cableways.—In some cases the topography does not lend itself to economical construction of parallel runways for two traveling towers. In such cases, it is generally feasible to locate a radial runway for a traveling tower on one side of the river, and a fixed high tower on the other side so that the cableway serves a fanlike area which covers the entire construction site. Typical installations of this type were used in the construction of Bonneville, Marshall Ford, and Conchas dams.

At Marshall Ford Dam one 8-yd radial cableway was used, but the other projects had two radial cableways, the two traveling towers in each case traveling on the same radial track. In most cases the operating machinery was in the traveling towers, thus making the fixed tower the tail tower, which at Marshall Ford Dam was 177 ft high, and at Bonneville Dam 223 ft high. The tail towers were designed with backstays to carry the cable pull applied at the top of the towers to deadmen anchored securely in the ground some distance to the rear. In this manner the towers acted as struts and little tension was carried in any of the legs.

The anchorage of the track cable and of the backstay cable deserves special attention in such installations since the tension in the track cable is in the

order of 200 to 250 tons. The problem is aggravated by the fact that with a concrete bucket traveling out and back in a 3-min to 4-min cycle, the release of load is fairly rapid (within 8 to 11 sec) and considerable bouncing of the track cable takes place due to change in load. In the aforementioned installations, this problem was handled in a very satisfactory manner by relieving the tower of most of the change in horizontal force and carrying such forces directly into the elastic backstays.

The connection at the top of the tower consisted of a pair of eyebars suspended from a horizontal pin. These bars were about 3 ft long and were free to swing in a vertical plane. The lower end of the bars carried a heavy pin, designed with a universal joint which in turn held the terminal eyebars of the main track cable. The universal joint permitted the track cable to move laterally through the arc made by the traveling tower at the other end of the span. A second pair of eyebars about 3 ft long was framed into the same lower pin and extended rearward to a connection with the backstays. In other words, the vertical eyebar hangars supported the juncture of the backstay with the track cable. Then the assembly was free to rock back and forth with changes in track-cable tension without changing the loading in the supporting tower over an excessive range, except in so far as a shifting of the resultant was represented by the swinging of the suspender bars. These eyebars were restrained to rock, only in a vertical plane, by means of rollers mounted at the outer end of the universal joint pin, and these rollers transmitted directly into the tower frame the lateral component that was introduced by the travel of the tail tower on either side of the center line.

Where the radial cableway covers a considerable arc, it may be advisable to carry two lines of backstays so that the track-cable forces are split into two components, with whatever internal angle seems most desirable.

The writer recently had occasion to explore the use of a backstay with the track cable anchored directly to the top of the tower without rocker links. Such a tower must either be strong enough as a cantilever to take the entire force or to carry part of the force within its range of deflection, the remaining force being carried by the backstay. For such a condition it was found that any kind of twisted cable or even locked coil cable has a modulus of elasticity (about 19,000,000) that is too low to permit much of the load to go into a backstay even when pre-stressed. In other words, the rigidity of the tower is ordinarily such that it tends to pick up most of the horizontal load within its normal range of deflection; and the resulting stretch in the backstays has not led to a substantial increase in stress even though the backstays have initially been pulled up until most of the sag in them has been eliminated. When it is desirable to use backstays in combination with a rigid tower, it appears advisable to make them of standard high tensile suspension bridge wires which have a modulus of elasticity of 29,000,000. This modulus is nearly the same as in the structural steel in the tower, and therefore they tend to work as a unit with the tower.

Mechanical and Hoisting Equipment.—A further interesting point with respect to arrangement of wheels on traveling towers is the drive for moving the

towers. The authors have shown, in Figs. 3 and 5, a system of cables anchored at each end of the runway with a spooling type of hoist mounted in the tower which can run back and forth on these cables to move the tower. More recently traversing drives have been developed consisting of motors and gearing—that is, direct-connected to the wheels on the rear trucks, similar to the drives on modern gantry cranes. Here again the localizing of principal forces at the rear of the tower simplifies the duty of such a drive in moving the tremendous weight of a cableway tower.

The main hoist described by the authors is the usual type of hoist and motor installation. An important advance in this respect was made on three of the large cableways for Shasta Dam, where variable voltage control was applied to large cableways for the first time. Such a control utilizes a motor-generator unit in each cableway, the generator being DC and furnishing power to a DC hoist motor. R. F. Emerson, electrical engineer for the General Electric Company, Schenectady, N. Y., has shown[11] that, with this system, the bucket can be accelerated or retarded smoothly by varying the generator voltage, and higher speed of the empty bucket is attained, thus reducing the time of the return trip over that required with AC drive. It was estimated that this arrangement would result in cutting 17 sec from the cycle time for each trip of the concrete bucket, and that during the life of the job about 580 hr of working time would be saved in the operation of each cableway. The total financial advantage in this case was evaluated at about $150,000. Also, the operator can spot the bucket with greater ease; the power factor of the entire system is raised and this results in a reduction in power charges; and the life of the operating cables is increased because acceleration and retardation are much smoother. The significance of these facts is appreciated by inspecting a graphical record[12] of power consumption taken during a typical cycle of operation on the Norris Dam cableway.

Economics of Cableway Installation.—Table 3 is of considerable interest because it shows that 68,000 tons of materials other than concrete were handled by the Norris Dam cableways during the construction of the dam. The availability of hooks of such high capacity to handle the great amount of other heavy materials and equipment has an important bearing on the economics of construction plant selection, and in determining the justification of cableways over other types of cranes for the placing of concrete.

A direct comparison from actual experience with respect to a large dam built by cableways, and a very similar dam built by whirler cranes, showed some interesting results. The monthly placing rate on each dam was, at its best, about 95,000 cu yd, or an average of 3,657 cu yd per day for the entire month. The best day on each dam was almost the same—namely, 4,500 cu yd.

The costs of those items directly influenced by the plant in the production and placing of concrete were as follows:

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[11] *Engineering News-Record*, June 6, 1940, p. 116.

[12] For illustration of graphic power chart, see "Construction Planning and Plant," by A. J. Ackerman and C. H. Locher, Members, Am. Soc. C. E., McGraw-Hill Book Co., Inc., 1940, p. 64.

Item	"Cableway" dam	"Whirler" dam
Plant for aggregate, cement, and mixing	$191,700	$157,800
Concrete delivery railroad	68,500	112,000
Cableways and appurtenances	328,300	0
Whirler cranes, trestle runway, and appurtenances	0	133,000
Total	$588,500	$402,800
Number of men on all operations	42	54
Plant amortization per cu yd of concrete on 1,000,000 yd	$ 0.589	$ 0.403
Operation, labor, materials, supplies, and maintenance per cu yd of concrete	0.709	1.274
Total unit cost	$ 1.298	$ 1.677

The difference in unit costs is $0.379, or $379,000 on 1,000,000 cu yd of concrete. It should be noted that whereas the first cost of plant for the "whirler" dam was practically 19 cents per cu yd less than for the "cableway" dam, the operating costs were substantially higher and resulted in a net difference of 38 cents per cu yd in favor of the "cableway" dam. The lower first cost of plant on the "whirler" dam, using standard equipment of smaller capacity, meant using smaller buckets, more hooks, more signalmen, more transfer trains and crews, more placing crews, more maintenance on account of more machinery, runway bridge maintenance, and extra placing costs around the legs of the embedded bridge, all of which contributed to increased operating costs in the production of concrete. This demonstrates the importance of spending enough money at the beginning of a job to obtain the lowest ultimate cost.

The writer recently had occasion to analyze in greater detail the relative performance of various cableway installations as well as other types of concrete handling equipment for most of the dams built during the fifteen years since 1925, and some important conclusions were derived therefrom with respect to economics of construction plant layout as well as speed of construction for various types of dams.

Tension Waves in Cables.—The tests described in the latter part of the paper are of special significance as they offer the first dependable information regarding stresses in operating ropes and track cables for large cableways made under actual operating conditions. The great increase in stress observed in these tests as compared with stresses computed by the usual formulas indicates the importance of dynamic conditions in cable systems. The sample chart in Fig. 16 is one of a number taken at various parts of the cableway. A close inspection of the chart indicates that the stress waves in the track cable occur not merely when a heavy load of concrete is suddenly discharged, but are present at all times while a load is traveling back and forth on the cable. The stress wave has a well-defined period of 5 sec or a frequency of 12 cycles per min. It is significant that this is a stress wave running axially in the cable

and not a bouncing of the cable itself. The most important conclusion from these tests was that the operating stress in the track cable is about 30% greater than that obtained by a conventional cable formula, because the cable formula is based on a static loading of the cable, whereas the increase in stress is due to dynamic loading.

In the spring of 1939, at Shasta Dam, the writer reviewed the proposed plans for a cableway layout of most unusual and ingenious form. This cableway layout consists of a fixed central tower with seven radial cableways all terminating in this tower. This scheme was devised by Francis Crowe, M. Am. Soc. C. E., and as a result of a great many studies of different types of construction plant and cableway arrangement, it was concluded that the adopted scheme would result in the concrete being placed at minimum cost and in the most practical manner. The central tower is a cantilever structure 460 ft high with a horizontal pull of 3,000,000 lb resultant at the top. This resultant is made up of a combination of loads introduced by six cableways with operating ropes, each of which develops a total pull of 400,000 to 450,000 lb. As a result of the tests made on the Norris Dam cableways, it was agreed that the applied loading on the tower for design purposes should include an allowance for dynamic loading at least 30% greater than obtains for static conditions. Considerable attention was given to the question of waves in cables and it can be readily visualized that with six or seven cableways, each carrying 8 cu yd of concrete and discharging at various intervals and sometimes simultaneously, a most complex condition of stresses is introduced at the anchorage in the top of the high tower. The spans vary in length from 2,600 ft to 720 ft and are arranged to cover the entire area of the dam within an arc of about 120°. The writer was tempted to christen this remarkable cableway arrangement "Crowe's Harp."

The phenomenon of stress waves in cables was observed more recently on a deep mine hoisting cable used in constructing one of the tunnel sections on the Delaware Aqueduct of the New York Board of Water Supply. It was found that the stress waves extended through the hoisting machinery into the 500-hp motor where they were observed by a graphical oscillograph which measured the change in magnetic field in one of the motor phase leads. The change in magnetic field was very pronounced, beyond any considerations given in the design of the motor, and might have caused serious damage if not detected in time. The period of the stress wave, as here measured electrically, coincided with an easily observed oscillation in the free inclined span cable between head frame and hoist.

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DISCUSSIONS

MEASURING THE POTENTIAL TRAFFIC OF A PROPOSED VEHICULAR CROSSING

Discussion

BY N. CHERNIACK, ASSOC. M. AM. SOC. C. E.

N. CHERNIACK,[8] Assoc. M. Am. Soc. C. E. (by letter).[8a]—In his discussion of the paper, Commissioner Van Wagoner has labeled it properly an "outline of attack upon the problem of determining the economic feasibility of a proposed vehicular crossing." It was intended to be just that. He has presented an excellent brief outline of the paper and has "put his finger on a weak spot," which, at the same time, happens to be one of the most difficult phases of estimating potential traffic. That phase relates to: (1) The determination of the "relative merit ratings" of all alternate routes on the basis of the present volumes these routes now handle along a number of "lines of travel"; (2) estimating the corresponding "relative merit ratings" of the proposed route on the basis of its travel characteristics relative to its competitor routes; (3) from these "ratings" determining the redistribution of travel, assuming the new route to be opened; and (4) thus estimating the shares which the new route would handle of each "line of travel." Commissioner Van Wagoner also justifiably complains that the writer has not explained why the rating equation should be in the nature of a compound interest or discount formula, nor how the value of d, the amount of the discount, is to be determined.

No economic law that is to be expressed by a mathematical equation can ever be induced from the observed quantitative data. The data may indicate the general type and "family" of curves that would best express the law mathematically, but the specific mathematical function must be selected arbitrarily by the investigator. After selecting such an equation, the data then will permit the investigator also to determine the "constants" in the equations by the "least-square" or other analytical or graphic method. It might also be pertinent to mention that the "least-square" method is limited to the de-

NOTE.—This paper by N. Cherniack, Assoc. M. Am. Soc. C. E., was published in February, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: June, 1940, by Murray D. Van Wagoner, M. Am. Soc. C. E.; and September, 1940, by Messrs. Charles B. Winick, and James H. S. Melville.

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[8a] Received by the Secretary October 14, 1940.

termination of "constants" in equations that can be converted into "straight-line" forms, by means of logarithmic, trigonometric, or other functions.

Thus, consider the particular problem at hand—that of establishing the law between the "relative merit rating" (or relative patronage) of a given route along a given "line of travel" and the several factors that motorists take into account in allotting their patronage—namely, running and waiting time, distance, tolls, and such other "preference" or "prejudice" factors as scenery (slums), safety (hazards), travel habits, etc. The simplest inverse relationship between the "relative rating" and the average inconvenience relative to the best crossing may be expressed by a straight line with a negative slope, as in the approximate Eq. 6*b*, in which the discount factor d is the constant slope in the straight line. Its "constants" may be determined as follows: Eqs. 6*b* and 7 are first combined into

$$r = 1 - \Delta t_r \, \Delta C_r \, d - \Delta t_w \, \Delta C_w \, d - \Delta D \, \Delta C_d \, d - \Delta T_d - \Delta C_p \, d \ldots . (12)$$

Eq. 12 being in the "straight-line" form, its "constants" may now be obtained by the "least-square" method by setting up as many equations as the data will permit. In each equation substitute the observed values of Δt_r (difference in running time along the "line of travel" via the given crossing and relative to the best route), Δt_w (relative waiting time), ΔD (relative distance), and ΔT (toll difference). Now solve by "least squares" to determine $\Delta C_r \, d$, $\Delta C_w \, d$, $\Delta C_d \, d$, d, and ΔC_p. Having determined d, solve for ΔC_r (the value of the running time difference), ΔC_w (value of the waiting time difference), ΔC_d (value of the distance difference), and ΔC_p (the combined value of the "prejudice" and "preference" factors). Of course, ΔC_p may be broken up into several individual "preference" or "prejudice" factors applicable to specific sections of routes, some of which may be particularly attractive and some particularly repulsive to travel. These values of the "constants" would constitute "first approximations."

Analysis of a great many "lines of travel," however, indicated that even the poorest alternative route carries some competitive travel. Hence, the "rating" of even the poorest crossing is practically never zero, and from its nature it can have no minus value. Therefore, a straight line with negative slope is only a crude approximation of the "rating" equation. Some curvilinear relationship would be a more realistic expression. A hyperbola, for example, might have been chosen as one family of such curves. The writer, however, chose the discount or compound interest curve, since it is a familiar everyday concept in financial transactions and lends itself to simple application to this problem. It may be translated to mean that the average motorist "discounts" the value of the route in the same way that a realtor would discount the value of a parcel of real estate—for example, relative to some other parcel of known value on the basis of factors which he knows affect real-estate values.

If Eq. 6*a* is adopted, therefore, it can first be readily converted to a straight line, logarithmically, as follows:

$$\log r = \Delta C \log (1 - d) \ldots \ldots (13a)$$

Then let $r' = \log r$ and $d' = \log (1 - d)$, so that

$$r' = \Delta C\, d' \qquad (13b)$$

Then by combining Eq. 13*b* with Eq. 7, the values of the "constants" may again be determined by the "least-square" method.

It will be seen that one's judgment need not enter into the determination of the values of the independent factors of costs of running time, waiting time, distance, and the values of the "prejudice" and "preference" factors or the discount factor "*d*," in the mathematical Eqs. 6*a*, 6*b*, and 7. These equations, therefore, are powerful tools in determining the values of these factors as placed by motorists in the aggregate at the time when, and the place where, the origin and destination surveys were made. On the other hand, the "constants" in these equations need not be determined strictly and rigorously by the "least-square" method. They may be determined by analytical, graphical, or trial-and-error methods, and even modified to any extent desired by the judgment of the investigator, arising from his intimate knowledge of conditions affecting specific routes. The equations as such, at the same time, picture clearly and accurately the manner in which motorists in the aggregate distribute their travel among routes of different attractiveness, as reflected by the several factors.

In his discussion of the paper, Mr. Winick was kind enough to state that the writer has at least rationalized the subject of traffic analysis. On the other hand, he proceeded at once to make a number of very serious criticisms as to the application of the principles presented, the most important of which, summarized, are that: (1) Future events are so much in the dark that the problem of measuring them is almost insurmountable; (2) the estimator of traffic and revenues of a proposed project is subject to greater pressure to raise estimates than the estimator of costs of construction to reduce costs; and (3) the "personal equation" is the predominant factor in estimating traffic and revenues.

In reply to the first criticism, it can be said that, on the one hand, the engineer is accustomed to taking account of future events, as in estimating future costs, for example. On the other hand, his projects also influence the future. In so far as the engineer controls a part of the future, he should be able to estimate its effects on his figures. Thus, it is now generally admitted that every new facility will "generate" some new traffic. In the past, however, engineers have "leaned over backwards" in estimating this item and, as a consequence, it has been invariably exceeded. In fact, in planning free highway facilities engineers may well be criticized for having either overlooked this item entirely, or having failed to give it sufficient value. The result has been that every new super-highway, by virtue of its tremendous superiority, has been swamped and clogged in the first few years of its life. For example, the writer is prepared to admit that traffic "generation" which results from the building of a vehicular project may be extremely difficult to measure. Its measurement, however, is not an insurmountable task by any means. It can be measured if sufficient data are collected and the necessary research to do it

is properly conducted. As in other problems of research, the engineer has the choice of "fight or flight."

In his second general criticism, Mr. Winick states that

> "In cases where the toll charge is determined * * * the most accurate traffic analysis will of necessity become prejudicial since all interests connected with the promotion of the project will tend to develop some one or another thesis that the required amount of traffic will be met or surpassed."

In effect, Mr. Winick thus accuses the traffic analyst of dishonesty under pressure.

Usually, traffic and revenue estimates for any given vehicular project are prepared simultaneously with estimates of costs of construction, maintenance, and operation. Subsequently, revenue estimates are balanced against cost estimates to determine economic practicability. In fact, it is often possible to make several approximations of anticipated traffic and revenues much before the corresponding cost estimates are prepared. These "horse-back" estimates indicate roughly, of course, the approximate annual costs of the project that the anticipated revenues could sustain. Under those circumstances, the cost estimating engineer could not be accused of cutting his cost estimates "under pressure" to meet the revenue estimates in order to get the construction program started. Professional integrity must be taken as an axiom if the engineer is to obtain the confidence of the public and the financier.

The third serious point that Mr. Winick makes is that, in the formulas prepared by the writer, so much engineering judgment is required that there would be a wide variation in the results calculated by different engineers. Therefore, it would be "difficult for a conscientious and honest analyst to avoid the danger of being somewhat optimistic," according to Mr. Winick, who suggests that "for that reason an unbiased analysis by a competent observer disinterested in the project is always desirable."

An independent review of any project is always desirable. That is the proper function and the contribution of consulting engineers to any project, especially of any magnitude.

That the formulas developed by the writer involve too much of the "personal equation" is a statement unsupported by facts. If, with the aid of the formulas, Mr. Winick is fearful of too great an optimism, how much more optimism would develop without any guide whatever! In fact, the formulas were developed by the writer to present some genuine restraining influence on the traffic analyst that has heretofore been lacking. On more than one occasion the writer has seen the pitfall illustrated by the following simplified example:

Assume two bridges, *A* and *B*, say 5 miles apart, tapping similar areas, the roads and approaches leading to them being about the same, and each, therefore, handling about equal traffic—say 5,000,000 vehicles annually. Now assume a proposed bridge, *C*, midway between and about equal in traffic characteristics and capacity to bridges *A* and *B*. In the computation of divertible traffic, it was assumed that since the proposed bridge *C* was about

equal trafficwise to bridge A, it would divert about half of A's traffic. Also bridge C being equal trafficwise to bridge B, it would divert about half of B's traffic. The divertible traffic of bridge C was thus estimated as 5,000,000 vehicles. This did not seem to be out of line, especially where the amount was integrated from the summation of a large number of "lines of travel." Under pressure, sometimes consulting engineers have not had sufficient time to check their conclusions to observe what traffic had remained on bridges A and B. Bridges A and B, of course, were left with 2,500,000 vehicles each, but bridge C was to handle 5,000,000 vehicles. Why? The three bridges are equally attractive. Should they not handle one third the existing traffic? ("Generation" is omitted at this point.)

By means of the rating formula developed by the writer, bridge A and bridge B each has a rating of 1 derived from its traffic volume. The share that each now carries is obtained either from its traffic directly or by dividing each rating by the sum of their ratings, each thus handling half of the total. Under the proposed conditions, the estimated rating of bridge C, being equal to either A or B, would also be 1. Its share would now be obtained by dividing its rating 1, by the sum of the ratings of the three bridges or 3, or a share of one third. The rating formula thus restrains honest optimism because it forces the engineer to abandon the concept that a new crossing merely diverts traffic from the existing crossings, to the concept that a new crossing produces a redistribution of traffic among the existing and new crossings. Furthermore, under the estimated redistribution, the formulas force the engineer to determine the traffic that each of the present crossings will be left with as well as the traffic that the new one will divert from the existing crossings. Under this concept, expressed by the rating formulas, reasonable and less optimistic judgment is obtained even without a quantitative application of the formulas. The purpose of any of the formulas proposed is not merely to substitute blindly the values of the "constants" and solve algebraically, but to give the engineer a quantitative concept of the phenomenon of distribution and redistribution of traffic flow under changed and changing conditions.

Table 11 shows a hypothetical case of traffic distribution under existing

TABLE 11.—HYPOTHETICAL CASE OF TRAFFIC DISTRIBUTION UNDER EXISTING FACILITIES AND A REDISTRIBUTION UNDER PROPOSED CONDITIONS

Crossings	UNDER EXISTING CONDITIONS			ASSUMING NEW CROSSINGS IN OPERATION		
	Annual vehicles (millions)	Percentage of total	Rating factor	Rating factor	Percentage of total	Annual vehicles (millions)
A	5.0	50	100	100	33.3	3.33
B	3.0	30	60	60	20.0	2.00
C	1.5	15	30	30	10.0	1.00
D	0.5	5	10	10	3.3	0.33
Old sum	10.0	100	200			
E	....	....	....	100	33.4	3.34
New sum	....	....	....	300	100.0	10.00

facilities and a redistribution under proposed conditions, assuming the new crossing E had the same rating as the best existing crossing A. The rating of existing crossings is assumed to be proportional to the volumes of traffic now carried along the given "line of travel" and the rating of the proposed crossing assumed to be based on its route characteristics.

It will be noted that the traffic redistribution, assuming a new one is opened, is of the present volume of traffic. This computation involves no estimates of any future increases or decreases at this point, but only a redistribution.

Rating of a Proposed Crossing Based on Cost Differences.—Mr. Winick attacks the values placed on running and waiting time in that here, too, some optimism may creep in; that the values are not constant; and that the resulting variations are too great to be useful.

In the first place, the term "constants" in the formulas does not mean values that are actually constant. That term refers to averages of variables whose variation is not too large. In the paper, the writer stated that monetary values would vary from time to time, from place to place, and under different conditions. The corresponding ratings would be affected in several ways.

Assume, for example, that in connection with one origin and destination study, Mr. Winick's "executive" and "young lover" were interviewed on their respective trips in each direction, that both had started from the same zone of origin and gone to the same zone of destination and had returned. Assume, also, that both went via the George Washington Bridge in the one direction but that on the return trip the "executive" returned via the George Washington Bridge and the "young lover," having only 25 cents left, had to return via the nearby competing ferry. Assume that both paid 25 cents more toll than that required by the nearby ferry and, in traveling over the George Washington Bridge, had saved 20 minutes. On the original trip, therefore, each paid 1.25 cents per minute saved. If there were only those four trips on that "line of travel," the result would show that of these four trips along this "line of travel," three trips, or 75%, were made via the George Washington Bridge at a cost of 1.25 cents per minute saved. The "executive" probably obtained a bargain rate via the bridge, although the "young lover" might also have paid more in order to make his date on time. How much more they would have paid per minute, they themselves would find it difficult to say unless they were actually faced with a situation of having to pay, say, 50 cents extra or $2\frac{1}{2}$ cents per minute. The "young lover" might even then have chosen the bridge to make his date. However, it is unnecessary to know how much saving in time is probably worth. The significant result is to determine what motorists have actually paid for convenience and to save time, etc., on a super-route, on the basis of the extra tolls that they had to pay.

In fact, in 1930, just such figures were prepared by the writer from origin and destination studies in that year to show the percentages of "lines of travel" which the Holland Tunnel was then handling, in competition with all competitive ferries combined as a group at varying costs per minute saved. (The George Washington Bridge was not opened until late in 1931.) This is a very simple way of expressing the results where there are only two crossings or, as

in this case, a tunnel and a group of ferries. As soon as a third or fourth crossing becomes involved, this method of arriving at the percentage handled by any given crossing, on the basis of time and cost, must be expanded into the rating formula (Eq. 5) since the percentage is affected by the number as well as the quality of crossings, expressed by the number and values of their ratings.

Furthermore, applying that very simple procedure of relating percentages of "lines of travel" handled by the Holland Tunnel at varying costs per minute saved, it was also discovered that percentages of some "lines of travel" were obtained by the Holland Tunnel in cases where there was no saving whatever in time or distance. Motorists paid 25 cents more than the ferry tolls, presumably on account of the extra convenience of the tunnel rather than by reason of the savings in time or distance. Hence, it appeared necessary to provide a "constant" to reflect this convenience or inconvenience, in connection with any given crossing. These "constants" were termed "preference" or "prejudice" factors. In some cases, the analysis revealed that the "preference" and "prejudice" factors stemmed not from the crossings themselves but from the travel conditions along the highway routes along the "lines of travel."

If motorists were actually stopped and questioned as to their "preferences" or "prejudices" for or against specific crossings or routes, some would and others would not be able to express the precise reasons; but the varying percentages of the total motorists, traveling along any given "line of travel," that prefer a given route is a subtle and infallible way of determining the mass motorist vote (based on extra tolls paid) as to "preferences" and "prejudices" of given routes. It is the writer's opinion (the contrary opinion of Mr. Winick notwithstanding) "that the same reasons under similar circumstances would motivate other motorists to use the new facility." If one cannot say this, one might as well "throw overboard" all research with respect to mass behavior of motorists.

Use of Empirical Formulas.—Mr. Winick remarks that

> "It takes but little observation to conclude that the annual increment of growth is dependent on too many physical changes to remain constant for any considerable period of years. Similar objections exist in any of the other empirical formulas expressing traffic growth."

The writer agrees but desires to refer back to his statement under the heading, "Measuring the New Traffic: Yardsticks of Traffic Expansion," in which he stated:

> "The foregoing empirical projections relate traffic growth to the mere passage of time and thus impelled the individual assumption of continual growth. They were thus manifestly defective. It was obviously more desirable to relate traffic growth to what might be termed 'traffic determinants,' which would permit anticipation of shrinkages as well as expansions in traffic volumes."

At the same time, it must also be noted that the traffic analyst must still rely, in many cases, upon empirical projections because he has not done sufficient research in any given locality to have developed better methods.

Relative Patronage Factors.—Mr. Winick states that "Closer analysis of relative patronage factors would indicate that they are not only psychological and economic in nature but geographical and social as well." This is quite true. It should be emphasized, however, that the monetary value of the factors, at the given time and in the given locality, will be determined from the studies of origin and destination of traffic in relation to the route characteristics. Such studies would show, for example, that time was worth more in New York than in Iowa. The writer never intended to leave the impression that the monetary values of time, distance, and other factors determined in New York should be applied in Iowa.

Trip Cost.—It is not the total or part of the cost of vehicle operation, but the monetary values that motorists place on route characteristics that determine their choice of routes. Trip costs, as used by the writer, represent not the usual cost of vehicle operation in connection with the trip (although expressed in cents per mile or minute), but the values placed by the motorist on the "resistances" of given routes over the standard, as determined from correlations between usage of routes, expressed in traffic volumes along "lines of travel," and route characteristics evaluated in dollars and cents. Even then the monetary values of time, distance, etc., are only used for weighting the effect of each of these factors to arrive at a composite single "resistance" factor, expressed in cents. Ratings of individual routes then vary inversely as the monetary values of route "resistances." The significant facts about these monetary evaluations are not the absolute values placed by the writer on the factors of distance, say, at 3 cents per mile, running time at one cent per minute, waiting time at 2 cents per minute, and preference and prejudice factors running from 5 to 15 cents, but the fact that, apparently, motorists, in the aggregate, consider a resistance of 2 extra miles of travel equivalent to a resistance of 6 extra minutes of running time, or 3 extra minutes of waiting time.

The writer has also observed that extra mileage as a "resistance" factor is apparently being given less and less weighting, while extra time, especially extra waiting time, is being given more and more weighting as a "resistance" factor.

"Generated" Traffic.—Mr. Winick uses the term "generated" in a different sense from that used by the writer, who defines "generated" traffic as that extra traffic which a new crossing develops in its first year of operation that would not have manifested itself on existing alternate routes if the new one were not opened. Any further expansion in traffic in the second and subsequent years is considered by the writer as part of the regular trend in traffic.

Traffic Determinants.—Mr. Winick states that

> "The determination of future traffic by such determinants as gasoline consumption, department store sales, payrolls, purchasing power, or what have you, is as good a method as is a forecast based on past performance; but even such determinants are subject to wide and unforeseen variation."

Such traffic determinants serve only to guide the traffic analyst in his visualization of the rate of expansion of the entire reservoir of traffic rather than

anticipated expansion in the traffic of any one facility whether new or old. It will be noted that, in Fig. 8, the writer used these determinations not in connection with traffic in the Holland Tunnel, or the George Washington Bridge, or the Lincoln Tunnel, but in connection with trans-Hudson traffic as a whole that included the traffic of not only these three trans-Hudson crossings but the traffic of all competing ferries, as well. Trans-Hudson traffic as a whole constitutes the traffic pool from which each of the individual Hudson River crossings will draw, depending upon their individual route characteristics along the "lines of travel" which they serve.

Mr. Winick further states: "In short, it would seem unsafe to forecast traffic for a newly proposed facility on such determinants." The writer again calls attention to his statement (heading, "Measuring the New Traffic: Economic Conditions"):

> "At the time vehicular traffic volumes are projected into the future, it is nearly impossible to foresee the probable changes in local economic conditions. These relationships, however, have served to explain the rate of traffic expansion in the past, and suggest that greater allowances in traffic trends will have to be made in the future for the effects of business conditions."

AMERICAN SOCIETY OF CIVIL ENGINEERS

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DISCUSSIONS

STANDARDS OF PROFESSIONAL RELATIONS AND CONDUCT

Discussion

By Messrs. Adolph J. Ackerman, and Daniel W. Mead

Adolph J. Ackerman,[42] M. Am. Soc. C. E. (by letter).[42a]—The engineers who can most appreciate Professor Mead's paper are those who, through a long record of high professional conduct, have experienced the many truths assembled in it.

As Professor Mead states in his "Introduction," the paper was written more particularly for the younger men of the profession. Each sentence might very well have been printed on a separate page because practically every one of them has enough substance to deserve a full page of discussion. The many principles are so fundamental that probably no one would attempt to take issue with them, yet how many really understand them, particularly among the younger and less experienced groups of engineers? The concept of "hard work" is quite different in the mind of the college student who is receiving all necessary financial support and enjoying the comforts of a well-furnished fraternity house, as compared with that of the student who is working his way by waiting on tables and doing miscellaneous chores, and who still succeeds in devoting enough time to intensive study to maintain a high scholastic standing. The same difference in concept applies to the many other professional ideals enumerated by Professor Mead.

Merely to hand out copies of Professor Mead's paper to students and young engineers is not enough. It would be unfair to expect of them a full understanding of the meanings and importance of those fundamental elements of character that go into a successful professional career. Such understanding can only be developed from personal contact with men of high professional standing who recognize their obligation of cultivating a closer master-apprentice

Note.—This paper by Daniel W. Mead, Past-President and Hon. M. Am. Soc. C. E., was published in January, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: March, 1940, by Messrs. Louis E. Ayres, Ivan C. Crawford, Walter H. Wheeler, Charles R. Gow, J. T. L. McNew, and W. L. Waters; April, 1940, by Messrs. Charles F. Scott, M. J. Evans, R. L. Sackett, Alonzo J. Hammond, A. B. McDaniel, C. B. Burdick, John M. Hayes, and G. W. Howard; and June, 1940, by Messrs. C. Frank Allen, Arthur W. Consoer, George C. Ernst, Frank S. Bailey, S. A. McCosh, John H. Meursinge, Karl W. Lemcke, John Sanford Peck, E. D. Ayres, F. E. Turneaure, Fred Asa Barnes, and C. A. Mead.

[42] Director of Eng., Dravo Corp., Neville Island, Pittsburgh, Pa.

[42a] Received by the Secretary October 4, 1940.

relationship. The young engineer invariably visualizes his own career in terms of that demonstrated by some admired senior in his profession and is influenced to strive for the same high standards of conduct.

When tempted to forget fundamental principles, one is most readily reminded of them by experiences in which those principles stood the test. Men derive their courage from the courage demonstrated by their predecessors. The new generation of engineers is entitled to have the gospel of professional conduct preached to them by qualified ministers of the profession, using Professor Mead's paper as a text. His paper therefore, is not only a challenge to the young generation but also to the old.

Daniel W. Mead,[43] Past-President and Hon. M. Am. Soc. C. E. (by letter).[43a]—The kind expressions of approval of most of those who have discussed this paper are appreciated. The writer's object in preparing the paper, however, was for the purpose of drawing out discussion of certain principles so that a code of conduct might ultimately be issued which would, in general, meet the approval of a considerable body of the members of the Society rather than to offer in any sense a finished code. He therefore welcomes such criticism as has been offered and regrets that more detailed constructive criticism has not been proffered.

The paper is criticised: (1) Because it is wholly unnecessary; (2) because it is too long and involved; and (3) because it is not sufficiently inclusive.

With the first criticism the writer is not able to agree and will answer more completely later in the discussion. With the other two criticisms he must, in part, agree. To the man of wide experience who, during his life, has thoughtfully considered the various principles of conduct, even a very brief statement of principles may be all that is necessary, and even such a statement may be superfluous. For the inexperienced man, however, a code should be sufficiently explicit to cover most of the new relations which he will have to encounter during the development of his professional practice.

On the other hand, in presenting this subject for the consideration of the profession, it is practically impossible to cover all possible engineering relations, and it therefore becomes necessary to include only the most common and important relations with the belief that the examples given will serve as a basis for their intelligent extension to the unusual conditions which will occasionally arise.

Mr. Waters states: "It should be sufficient to say that 'every engineer should at all times think and act in accord with the highest principles of personal honor.'" It might also be suggested that the Golden Rule "Do unto others as you would have others do unto you" is all that is needed in such a code; and this is true in both cases if the individual is capable of applying either rule under the manifold conditions under which such rules should be applied.

This is the weakness of all brief rules of conduct. In themselves they contain no information as to how they should be applied in detail to professional

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[43] Prof. Emeritus, Hydr. and San. Eng., Univ. of Wisconsin; Cons. Engr., Madison, Wis.

[43a] Received by the Secretary August 14, 1940.

conduct which, in the writer's judgment, is a serious mistake if they are offered to students or young engineers to guide them in their professional work. The writer believes that in the teaching of the application of the best principles of conduct the rule must be as specific as possible so as to apply as nearly as practicable to specific conditions.

The writer agrees with L. E. Ayres that it would be well to include a section covering the conduct not only of the expert witness but also of the appraiser and the engineer who make reports on the feasibility of proposed new engineering structures and plants, and especially when such plants and structures are to be financed publicly. He believes that the principle designated as No. 8 by Mr. Ayres (or its equivalent) might well be added to the official code of the Society (29),[3] and also as Paragraph 10 to Section III in the paper.

The writer agrees with Mr. Waters that "It is youth which must preserve the high ideals and standards of honor"; yet American schools are annually graduating thousands of young men from their engineering courses who have had no instruction in these high ideals and standards of honor or in professional conduct. Almost all universities, colleges, and schools are derelict in not affording instruction in this subject to all students.

The writer's experience is quite parallel with that of Professor McNew. Almost every year some young engineer from almost every school fails to abide by his agreement to accept a position offered at an early date and accepts instead a later offer at a higher salary. In this way the student's first action at the threshold of his professional life is unethical because of the financial advantage to him. This desire for financial gain is directly or indirectly the cause of most unethical conduct in professional work.

The fact that an older engineer of good reputation has not always acted in accordance with the principles of ethical conduct is not a sound basis for the similar conduct of others. The writer fears that there are no engineers in practice today who, looking back on their professional lives, can approve every act of their own professional conduct; and the personal acts which they now condemn are commonly the mistakes which they made, through ignorance or thoughtlessness, in the early days of their professional lives.

In answer to the comments of Mr. Howard, certain principles were repeated in more than one section in order to make each section more nearly complete in itself.

It will have to be admitted that the percentage method of evaluating "characteristics necessary to success in engineering" (Table 1) is inexact and faulty, but no other method seems available to illustrate the opinions of the profession as to the importance and essential value of attributes not commonly emphasized in engineering as now taught. Most professional men will agree as to the essential value of these characteristics if not in their percentage rating.

In reply to Mr. Consoer, the writer would call attention to Section IV, Paragraph 2, in which he states: "Honorable competition for promotion and opportunity for employment is an essential part of modern democratic civilization." The first sentence in Section IV, Paragraph 19, is simply an expression

[3] Numerals in parentheses, thus: (29), refer to corresponding numbers in the Bibliography; see Appendix.

of the writer's idea as to the ideal way in which employment should be awarded. In the latter part of Paragraph 19 the writer expresses disapproval of unprofessional methods of competition which are all too common at the present time, and which he believes most practicing engineers also deprecate.

The writer was surprised at the principles advocated by Professor Peck. Such principles are unsound and dangerous, especially if presented to young men during the period when their professional and ethical attitudes are being formed. Professor Peck states that he is "opposed to fixed, rigid codes * * * be they religious, moral, or professional." Does his opposition include the legal codes of the federal and state governments?

Laws are but rules of conduct established by legislative action and enforced by the courts. These laws are supposed to represent the views of the majority of the people in the community in which they are established as to what individual action should or should not be allowed. There are thousands of these laws on the federal and state statute books; and there is scarcely one of them that under certain specific conditions may not be unfair and unjust to some individuals. Such laws frequently become archaic and are repealed or, if negligently left on the statute books, their absurdity becomes apparent and embarrassing, and public opinion demands and secures their repeal. The fact that the world is changing does not destroy the necessity for law, but it does necessitate its readjustment from time to time as conditions develop.

It is a fact with which every one is familiar that an individual may strictly observe the laws of the land and yet be an undesirable citizen and a poor neighbor. The idea that each individual can and should establish for himself rules of conduct for such relations as are not covered by law and without reference to the experience or opinions of others seems equally as absurd as would a similar attempt to establish principles of law. Laws must be established by the majority action of a legislative body, and rules of professional conduct must be based on the concurrent opinions of the members of a profession.

It is true of established rules of conduct, and also of rules of law, that few ethical or legal principles are universally applicable, and that in certain cases each individual must depend upon his common sense and conscience as to what his conduct should be under the limitations of the conditions under which his conduct must be exercised.

Common sense and conscience are the results of early training, of the personal influence of those with whom one comes in contact, of education, of experience, and of such reflection as the individual may give to these factors. On this account common sense and conscience are limited in their application to the breadth of the experience on which they are based, and can be applied successfully only within such limits. When that experience is extended, common sense and conscience are sometimes of benefit by analogy but are very often apt to be mistaken because of the limitation of experience.

A young man leaving college is plunged frequently into a life so different from his previous experience that he needs the advice of the men who have already had the experience that is about to become his. Rules and principles are then of value in reflecting the conclusions of those who have gone before.

Just as precedent in the design and construction of engineering structures is of value when similar structures are being designed and constructed, so principles of conduct established by the experience of one's predecessors are of value in the consideration of one's own line of action. The writer will agree that precedents are not to be blindly followed; but to ignore the experience of others, either in engineering design or in conduct, is a dangerous and often a serious mistake. Regarding this point, Raymond Moley observes:[44]

> "What are principles that men live by? What, for that matter, is the meaning of principle itself? Reduce the question of principle to the case of an individual and one of the problems he faces. A man does not govern his life by chance. He learns, as the years pass, and profits by what he learns. He learns that there are some things that he cannot eat without distress. He learns that there are some games he cannot play. He learns that there are ways of doing his work better. He learns how to conduct his relations with other people. Out of the accumulation of individual experience he creates rules for himself. As time goes on, those rules become principles of living. He finds that by observing and respecting them he saves himself untold trouble and discomfort. He doesn't have to argue out thousands of individual decisions with himself. He depends upon his principles. Ultimately he lives not only with but by them."

It is unlikely that a man of advanced age and long experience would, in his ordinary relations in life, find it necessary to study a code of conduct in order to determine what his own conduct should be; neither would he ponder very deeply concerning the various possible outcomes of his action. His answer would be given at once, based on his established principles, and would probably be correct. If, however, the conduct concerned new conditions entirely beyond and different from his previous experience, then comes the necessity for due consideration; and in such cases the opinion of those who have had similar experiences and have reached definite conclusions cannot safely be ignored.

A person does not seek legal advice concerning his ordinary conduct in everyday life, but when he enters into new legal relations with which he is not familiar it is the part of wisdom to seek advice.

The Golden Rule is, and should be, in general, the basis of almost all ethical conduct. The Golden Rule is as sound today as it was in the days of Christ. It may puzzle the philosopher who desires to live by the Golden Rule as to what he should do if he captures a thief who is attempting to plunder his home. However, the practical man who also desires to live by this rule would recognize his obligations to society and would at once hand the malefactor over to the police.

So far as the writer's information goes, a code of ethics was first established for physicians about 400 B.C. by Hippocrates, a famous Greek physician often called "The Father of Medicine"; and the writer understands that Hippocrates' "oath of service" is still used in some medical schools. This would seem to indicate that a "changing world" has not seriously affected the code of Hippocrates in more than twenty-three hundred years. In a similar manner, although certain purely technical requirements of a code of conduct may

[44] "Indispensable Principles," by Raymond Moley. An address delivered on Constitution Day—September 17, 1940—to the Union League Club of Chicago.

change with changing conditions, the fundamental principles of good conduct and good ethics are unchangeable and eternal.

Legal ethics is of many years standing. Today thousands of codes have been adopted by professional societies, technical and business organizations and associations, and numerous books have been published discussing the basis of business and professional relations contained, or those which should be contained, in these codes. Is the assumption warranted that all this effort is wasteful and unnecessary and that each individual in all these diverse interests is qualified to evolve from his own reflections all of the principles of conduct which he should exercise under each of the thousands of circumstances in which he may happen to be or into which he may possibly enter from an entirely different environment? The writer cannot but feel that such a proposition is an absurdity.

Due consideration of what is best for all is both desirable and essential. However, the writer has never seen the "edge of the moral responsibility" of any man so keen that he thought it would be dulled by the reading of a code of conduct or by consulting the opinion of those who have had previous experience in an unknown field into which the individual was about to enter, whether such field be in engineering or in conduct.

There is no doubt that there are many who are mentally incompetent and that in many cases failures are due to such cause. However, lack of ambition and mental laziness are commonly ascribed to such cause. If and when psychological findings become uniformly reliable, it will perhaps become desirable to limit the efforts of students to those fields within their abilities. In the meantime, however, the likes and dislikes and the ambitions of the student are the general guide to his line of efforts and he should not be discouraged by imaginary inhibitions, etc. It might have been better if this matter had been discussed in greater detail in the paper somewhat as follows:

It is very desirable for every man to realize that there are great variations in the normal character and ability of men as well as in the opportunities which occur. These factors necessarily must have a very important effect on their possible development and the degree of their professional success. The higher positions in every line of professional and business life are comparatively few in number when the great number of professional aspirants is considered; and it is only those few who combine high character, great ability, and favorable opportunities who can possibly hope to acquire one of those outstanding positions.

While it is the part of wisdom that each young engineer should recognize that he is probably only an average man, it is well for him to remember also that such positions frequently have been filled by men who have, in early life, occupied the most humble positions. It seems desirable also that each man recognize that careful attention to his own character and progress, and an intelligent and strenuous attempt at the best possible development of himself, will accomplish the most that can be expected under his own individual circumstances.

A strong personal attempt at self-improvement and a personal interest in his work will result not only in the best that each one can hope to accomplish

but will assure pleasure and happiness in each man's work, as well as contentment in the position which he will attain regardless of whether or not he attains the heights of ambition to which in his youth he aspired. To do the work which he undertakes in a thoroughly satisfactory manner will usually give more personal pleasure and satisfaction and a greater feeling of personal success and happiness than would a position of greater responsibility and more arduous demands which the individual may be poorly equipped to occupy.

The writer can appreciate the experience of Prof. E. D. Ayres with his students. In many years in teaching Engineering relations to senior engineers, the most common inquiry received by the writer was perhaps how to do an unethical thing in an ethical manner. The action usually involved was the breaking of an agreement to accept employment, which agreement was made months before graduation, in favor of accepting a more remunerative job offered just prior to graduation. Although the writer never was able to give an answer satisfactory to the student, he always advised such students—if they had decided to break their first engagement—at least to request release from the earlier commitment as a more manly course than bluntly advising his first would-be employer that he had taken another and better job. The question asked Professor Ayres as to "how to chisel the other fellow" has been commonly answered in the words of David Harum: "Do the other fellow the way he would do you, and do him first." This has been applied not only in horse trading but also in other lines of business; but it is far from good ethics for the business man, and especially for the professional man.

Section IV, Paragraph 17, does not apply to a graduate engineer seeking a job with an employer who has full knowledge of his status. Both Paragraphs 17 and 19 should not apply to graduate engineers, and they do not unless such graduate contemplates trying to enter general practice shortly after graduation. Such an action in itself should certainly be considered highly unethical as, under such conditions, the young engineer—as a rule—can have little of value to offer clients, and, as a result of his inexperience, his professional work would be unsatisfactory to his client and would injure that young engineer's reputation and prospects.

General practice should never be attempted until the engineer has acquired years of experience and can give value received for his compensation. It should be impressed on all engineering students that they are not engineers upon graduation but become engineers only after they have learned to apply correctly, to practical engineering problems, the principles of science and the technical principles and rules learned in school. It is true, of course, that some individuals acquire common sense and ability to apply principles to practice much earlier than do others, and that length of service is not always a sound basis for judging the relative ability of several men.

Finally, the writer desires to make certain corrections in the text of his original paper in order to make its meaning more certain:

In Section IV, Paragraph 6, line 1, after "prepared" insert "especially for him"; Section IV, Paragraph 19, line 3, change to read "the medical and legal professions. This is the end to be sought. Honorable competition is often necessary under present conditions, but solicitation by lobbying, the"; in

Section V, Paragraph 5, line 2, after "failure" insert "of purpose"; and in Section VII, Paragraph 6, line 2, after "responsible" add "qualified."

He also desires to add to the principles given the following:

To Section VII, as Paragraph 9. "No work should be advertised for bidding until definite and complete arrangements have been made for financing; otherwise contractors may be put to unjust and unnecessary expense."

To Section I, Paragraph 15. "Truthfulness and dependability are the fundamental basis of all proper conduct. The young engineer should agree to accept a position only after he has fully made up his mind to carry out his agreement. His failure to do so on account of a later offer at a higher salary is unethical and therefore a very unsatisfactory way to begin work in the profession."

To Section III, Paragraph 10. "It is unethical to give expert testimony or to advise his client in a manner differing in substance or implication from what he would present and defend before a meeting of this Society."

Paragraph 11. "It is unethical to submit a proposal, or to enter into contracts, for the construction of work, for which plans and specifications have been prepared by him as engineer for a client."

Paragraph 12. "It is unethical for an engineer to be associated in the conduct of professional work with others who do not conform to the standards of this code." This makes necessary the following change in the discussion by Louis E. Ayres in March, 1940, *Proceedings:* Page 572, line 21 to read—"The writer is an advocate of the idea expressed by Section III, Paragraph 10, having to do" etc. Likewise, in the discussion by E. D. Ayres in June, 1940, *Proceedings*, page 1157, delete the last paragraph; and page 1158, delete the first paragraph.

In closing this discussion the writer desires to state that this paper was prepared for three separate but similar reasons.

First: In his college work the writer does not remember that he ever received any material information or instruction on Professional Relations or Professional Conduct. He was obliged to gather such information in the expensive school of experience and "hard knocks." He has always felt that he might have done much better work and made many less errors if there had been available during that period the experience and conclusions of those who had already trod the difficult road to successful professional practice. He has felt, therefore, that it was incumbent upon him to make available to the younger men of the profession the conclusions which he has reached after more than fifty-six years of professional practice.

Second: Early in his professional practice the writer's attention was drawn to the unfortunate reaction of certain young men who, after leaving college, were thrown under the influence of certain men whose practice was never to subordinate profit to high ideals. He witnessed the fact that under such influence these young men seemed to assume that the actions they observed must be common to their profession; and such actions they too often adopted in their own conduct. The writer then believed, and still believes, that some advice given during college training of these young men would in many cases have afforded a means of avoiding the mistakes made by them in the early

days of their experience. For this reason, when the writer became a teacher in an engineering school, he gave some time, in all the courses which he taught, to instruction covering the proper engineering relations in professional life; and in his classes in Contracts and Specifications the writer spent considerable time discussing this important subject. The reaction of his students who have volunteered their opinion of his efforts, after many years in practice, has convinced him that at least in many cases the discussion of these subjects has been of material benefit to many engineering graduates.

Third: During his experience as a teacher, the writer had occasion to read many professional codes of ethics and numerous books written on the subject, in order to secure proper material for the presentation of this subject to his classes. He found that most adopted codes had evidently been written for older professional men in general practice and covered only a very limited field for the younger men who occupied the lesser positions in the engineering field. The writer is thoroughly convinced that, if an advisory code of professional conduct is available and is called to the attention of the younger men during the formative period of their professional training, it will be of material assistance to many of them. He believes that most engineering students desire to do the right thing and will, in their professional lives, follow the habits which they have formed in their youth in playing their college games, of following the rules of the game in playing the most important game of life.

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DISCUSSIONS

MASONRY DAMS
A SYMPOSIUM

Discussion

By Messrs. Berlen C. Moneymaker, A. Warren Simonds, and W. J. E. Binnie

Berlen C. Moneymaker,[94] Assoc. M. Am. Soc. C. E. (by letter).[94a]—Mr. Crosby presents an interesting discussion of the more common geologic and foundation problems of masonry dams founded upon rock, and illustrates these problems by specific reference to successful dams and dam failures on various types of rock and geologic structure. Although these problems are discussed more or less fully in numerous excellent papers by Charles P. Berkey,[95] M. Am. Soc. C. E., Kirk Bryan,[96] Chester K. Wentworth,[97] L. C. Glenn,[98] Warren J. Mead,[99] Affiliate Am. Soc. C. E., and other geologists, Mr. Crosby's treatment of them will be of considerable interest to engineers.

The thesis of Mr. Crosby's paper—that essential foundation conditions can be determined in advance of construction, that safe and successful dams can be built at most sites, and that dam failures are not entirely unavoidable—is now rather generally accepted by both engineers and geologists. However, the geologist must bear in mind that geology is not the only consideration involved in the selection of a dam site. In programs requiring the integration of several projects, the location of a given dam may be fixed within a very short stretch

Note.—This Symposium was published in May, 1940, *Proceedings*. Discussion on this Symposium has appeared in *Proceedings*, as follows: September, 1940, by Messrs. William P. Creager, J. R. Shank, George R. Rich, Robert A. Sutherland, Ross M. Riegel, Paul Baumann, W. A. Perkins, L. J. Mensch, and Lewis H. Tuthill; and October, 1940, by Messrs. F. A. Nickell, Leslie W. Stocker, Barton M. Jones, P. E. Gisiger, Joseph A. Kitts, S. O. Harper, and R. F. Blanks.

[94] Asst. Chf. Geologist, TVA, Chattanooga, Tenn.

[94a] Received by the Secretary September 13, 1940.

[95] "Responsibility of the Geologist in Engineering Projects," by Charles P. Berkey, *Technical Publication No. 215*, A.I.M.E., New York, 1929, pp. 4–9.

[96] "Problems Involved in the Geologic Examination of Sites for Dams," by Kirk Bryan, *loc. cit.*, pp. 10–18; also "Geology of Reservoir and Dam Sites," by Kirk Bryan, *Water Supply Paper No. 597*, U. S. Geological Survey, 1929, pp. 1–38.

[97] "Geology of Dam Sites," by Chester K. Wentworth, *Technical Publication No. 25*, A.I.M.E., New York, 1929, pp. 78–96.

[98] "Geology of Dams and Reservoirs," by L. C. Glenn, *Technical Publication No. 215*, A.I.M.E., New York, 1929, pp. 97–110; also "Geology Applied to Dams and Reservoirs," by L. C. Glenn, *Proceedings*, Engrs. Assoc. of the South, Vol. 26, 1915.

[99] "Engineering Geology of Dam Sites," by Warren J. Mead, 2d Cong. on Large Dams, Question VI, Washington, D. C., 1936; also *Civil Engineering*, May, 1937, pp. 331–334, and June, 1937, pp. 392–395.

of the river which does not afford a wholly satisfactory dam site. In such cases, it is the duty of the geologist to locate the best site available, under the restrictions imposed, and to discover and to acquaint the engineers with all the geologic conditions which must be met by design or foundation treatment, or both.[100]

Perhaps the outstanding feature of Mr. Crosby's paper is his outline of the stages and objectives of a complete geologic investigation of a dam site. The geologic investigation of dam sites by the Tennessee Valley Authority (TVA) is conducted along lines almost identical with those outlined.

The geologic work of the TVA is done by the Authority's own highly specialized staff, and reviewed periodically by geologists on the Authority's board of consultants. When a dam is seriously considered for a certain stretch of the river, a geologic field party is detailed to the locality. This party consists of a well-trained engineering geologist with considerable experience, and his assistant, also well trained in geology but with less experience. The geologic work done at a TVA dam falls into the following stages: (1) Preliminary investigation, (2) exploration, and (3) construction.

(*1*) *Preliminary Investigation.*—This investigation embraces more than a brief reconnaissance study. It includes studies of the physiography, stratigraphy, and geologic structure of the area involved, as well as studies of the character and thickness of the overburden. In the course of this work, a geologic map of the area, showing in detail the various types of rock, contacts, faults, and joint patterns, is prepared on a scale of about 1 in. to 500 ft. Throughout this stage, very close cooperation is maintained between the geologist and engineer, so that the site finally selected for exploration is the one that best meets all requirements.

(*2*) *Exploration.*—The geologic work in the exploration stage involves the development of the details of structure, lithology, and rock conditions. By means of drill holes, trenches, pits, and tunnels, the configuration of the top of rock is determined, the overburden is classified, and the extent of rock weathering and cavitation is determined. Numerous geologic sections are drawn to show the structure, lithologic units, cavities, and zones of rock decay in as much detail as possible along various ranges. Very close cooperation between the geologists and engineers is continued throughout the exploratory stage.

(*3*) *Construction.*—A geologic field party is maintained on each project throughout the construction period. In this stage, hundreds of large-scale drawings are made, showing in detail all the geologic features that have any bearing upon engineering problems. These drawings, supplemented by numerous daily discussions with the geologist, keep the engineers informed in advance of all conditions to be met. In no stage of a project is the geologist more useful than in the construction stage.

Although a geophysical party was maintained, for a while, by the Authority's Geologic Division, it has been found desirable to limit the work of the geologist to geology, leaving the work in the allied sciences to specialists in those

[100] "Bad Rock Limits TVA Dam Location," *Engineering News-Record*, October 21, 1937, p. 665.

fields. Thus, all work in soil mechanics is done by experts in this field; the actual testing of materials is left to materials testing experts, etc.

Mention is made by Mr. Crosby of the usefulness of petrographic studies of the rocks involved in dam sites. The writer has found this method of study of considerable value, especially in areas of volcanic rocks. Many of the basic igneous rocks—basalts, andesites, basic tuffs and agglomerates, and diabases—contain hydrous silicate minerals of the zeolite family as veinlets and cavity fillings. As the zeolites are unstable and undergo volume changes in response to changing conditions of humidity, they can effect the complete disintegration of the enclosing rock in a very short time. The appearance of rock containing these minerals is often deceptive. Rock that is hard and apparently sound may go to pieces in a few days after its exposure. Fortunately, however, a petrographic study will reveal in advance whether zeolites are present in amounts sufficiently large to be dangerous.

Another geologic feature that may affect a dam (especially a high dam) adversely is sheet jointing or "sheeting planes." Massive rocks of igneous or metamorphic origin are likely to be divided into sheets by joints developed parallel to the topographic surface. Although the origin of these structures is perhaps debatable, it is generally attributed to relief of stresses as a conse-

FIG. 27.—RIM LEAKAGE AT GREAT FALLS DAM, IN TENNESSEE

quence of unloading by erosion. In the abutments at a dam site, the sheet joints are inclined toward the river; in the rock underlying the river they are horizontal or inclined slightly in a downstream direction. These structures are generally marked by zones or "seams" of permeable decayed rock and, unless they are removed from the foundation or sufficiently treated, may impair

the success of the dam by facilitating leakage or by resulting in uplift or settlement. At Hiwassee Dam, lenses of rotten rock developed along sheet joints as thick as 3 ft alternated with thinner or thicker layers of sound rock to a depth of more than 50 ft below the bed of the river.

Mr. Crosby cites the enormous leakage at Hales Bar Dam on the Tennessee River as an example of what is likely to happen at a dam built without a geologic investigation. That a geologic investigation does not necessarily insure the success of a dam is strikingly attested by another masonry dam on flat-lying and cavernous limestone in Tennessee. The Great Falls (Rock Island) Dam on Caney Fork River had the advantage of geologic investigations by three reputable consulting geologists of very broad experience. Reports furnished the TVA show that these gentlemen recommended the site as suitable for a dam 110 ft high. The dam was built to a height of 35 ft in 1916, and in 1925 it was raised to 60 ft—50 ft lower than the height recommended. After the dam was raised, leakage appeared through the abutments and through the narrow rim that separates an arm of the reservoir from the river gorge below the dam. As the water short-circuiting the dam washed out clay seams and cavity fillings, an increasing quantity of leakage developed through the reservoir rim, producing a series of spectacular waterfalls downstream from the dam (see Fig. 27). A much greater portion of the flow of the river at this project is lost through leakage than is the case at Hales Bar.

Although other instances like Great Falls could be mentioned, there can be no doubt that the geologist of today is contributing more to the success and safety of dams than the geologist of a decade or so ago. Moreover, the geologist of today, who remains at a project from the initial exploration through the construction stage, contributes materially to the saving of money on foundation preparation and foundation treatment.

A. Warren Simonds,[101] Assoc. M. Am. Soc. C. E. (by letter).[101a]—In connection with the design of contraction joints, Mr. Steele has discussed the spacing of the joints, types of keyways, and the metal sealing strips for holding the fluid grout within the joint until the cement has set. In the construction of contraction joints, a good design may become of questionable value due to faulty construction at the metal seals or by the plugging of essential header pipes. Only too often do contractors consider the items used in contraction-joint construction to be merely pay items which, after being installed, are of no further value. This is usually true where the placement of large quantities of concrete is involved. Rigid inspection in such cases is essential.

In grouting vertical radial joints in a high arch dam in a narrow canyon, it becomes necessary to do the grouting in lifts. In such cases it has been observed that the grouting of every lift tends to open a V-shaped opening in the lift below, thereby splitting the bond of the grout film and preventing the blocks of the dam from having a complete bearing. In an attempt to remedy this difficulty a 3-in. horizontal cutoff groove has been tried in place of the horizontal sealing strips. This would permit the regrouting of a lift while grouting

[101] Engr., U. S. Bureau of Reclamation, Denver, Colo.

[101a] Received by the Secretary September 10, 1940.

the next lift above. From the standpoint of the grouting operations, the cutoff groove was not as satisfactory as the sealing strip because the grout tended to run by the groove and foul the grouting system of the next lift above. Where artificial cooling of the concrete is used, present practice is to use horizontal sealing strips and to subcool the concrete prior to grouting. As the joints are grouted at the time when the temperature of the concrete has reached the minimum, the subsequent expansion of the concrete due to rising temperature would tend to close the open spaces which may possibly exist within the upper part of the joint.

In designing the grout system for an arch dam, the height of the grouting lift should be governed by the stability of the blocks when the joints are subject to grout pressure. The upstream component of the grout pressure in radial joints may cause tensile cantilever stresses of considerable magnitude at the downstream face of the blocks. The usual heights of grouting lifts range from 50 to 100 ft where grout pressures of 25 to 50 lb per sq in. are used at the top of the lifts.

The ideal joint grouting job would be one which permits the simultaneous grouting of all joints in the dam for their full height, as stated by Mr. Steele. As this procedure is rarely possible, due to the dimensions of most dams, the logical procedure is to grout all the joints up to the same elevation at one time. This can be done by connecting the headers of the joint grouting systems to a common supply line from the grout pump. Known quantities of grout can then be injected into each joint in rotation, thereby maintaining the surface of the grout at approximately the same elevation in the joints. This tends to reduce the unbalanced tipping of the blocks in an axial direction to a minimum. Filling one joint at a time is likely to cause an excessive opening of the joint being grouted, while the adjacent joints may be squeezed shut.

In grouting the joints of a large dam, it is not feasible to grout an entire lift of the dam from abutment to abutment in one operation. Groups consisting of 12 to 36 joints are therefore grouted as units. The grout pressure in the boundary joints is balanced by holding water under pressure in the adjacent joints. This pressure is regulated so that the closing of the adjacent joints is controlled until after the grout is set.

Control of the grouting operations should be governed by careful observation for the deflection of the blocks while grouting. Dial gages registering to 0.0001 in. are valuable for mounting across the contraction joints to observe the spreading of the joints. Whitimore type strain gages can also be used for this same purpose. Deflection at the top of the blocks can be observed with a theodolite. The magnitude of the deflection of the blocks of a dam depends on the dimensions of the blocks. In general, the opening of a contraction joint due to grout pressure should not be greater than 0.02 in. at the top of a lift having a height of 50 ft. Undesirable tensile stresses may develop if greater spreading occurs. Upstream radial deflections of the blocks varying from $\frac{1}{4}$ to $\frac{3}{8}$ in. have been observed while grouting the joints of arch dams having heights of approximately 200 ft.

The number of joints that can be grouted in one operation depends on the equipment and organization available. A total joint area of 100,000 sq ft

(neglecting keyways) can be grouted in an 8-hr shift using one 10 by 3 by 10-in. duplex grout pump and one grout mixer having a capacity of about 20 cu ft. Slow grouting is desirable in injecting the grout. The length of time required to fill one large joint may be as much as twelve hours. Where the temperature of the concrete is low, the grout will remain fluid longer than when the temperature is high.

The properties of cements used in contraction joint grouting vary widely. Standard portland cement, high early strength cement, modified cement, and slow setting oil well cement have been used. Cements purchased under the same specifications vary widely when supplied from different mills. In general, an air-separated cement is to be preferred, although an extremely fine-ground, air-separated cement may become lumpy before use, due to "warehouse set." The mechanical screening of cement at the job will produce the best cement for contraction joint grouting although its cost will be relatively high. Such a cement can be delivered to the mixer free from lumps, unground clinker, and tramp iron. A good cement for contraction joint grouting should have a fineness such that 100% will pass the 100-mesh U. S. standard screen and 95% will pass a 200-mesh screen.

A contraction joint grouting system is a fairly costly feature of dam construction. For best results, care should be used to effect an adequate job of grouting. The joints should be thoroughly cleaned by washing with water and compressed air. The amount of muck from construction that may accumulate in a contraction joint is often surprizing in quantity. After washing, the joints should be soaked with water for twenty-four hours immediately prior to injecting the grout. The consistency of the grout used varies from 2 parts of water to 1 of cement by volume to 0.6 part water to 1.0 of cement in extreme cases. The usual mix ranges from 1 : 1 to 0.7 : 1 of water to cement.

Reliable and suitable equipment is essential in joint grouting. In order to reduce the hazard of equipment failure, it is desirable to provide a standby unit which can be put in operation in the event of an emergency. An extra grout pump should be considered a standard part of the equipment. Lubricated plug valves on the supply headers have proved the most satisfactory for use with grout under pressure. Use of this type of valve reduces the sticking and binding so often experienced with cheap valves.

This Symposium presents the most recent assumptions and theories involved in the design of high and important masonry dams of the single-arch, curved gravity and straight gravity types. Much of this information has been obtained from experience in the design and construction of some of the world's largest dams. The paper by Mr. Steele is a necessary part of the Symposium as it covers the treatment of construction joints in mass concrete. This paper contains some much needed information for engineers engaged in the design of concrete dams and also tables of data which serve as a record of the design and method of treating construction joints.

W. J. E. Binnie,[102] Hon. M. Am. Soc. C. E. (by letter).[102a]—Messrs. Houk and Keener are to be congratulated on the lucid manner in which they have

[102] Civ. Engr. (Binnie, Deacon & Gourley), Westminster, S. W. 1, London, England.

[102a] Received by the Secretary September 24, 1940.

dealt with their subject, but they make certain assumptions for the purpose of computation which the writer considers are open to question. Assumption 3 deals with the foundation rock which must be assumed to be homogeneous and uniformly elastic in all directions, a condition which Messrs. Houk and Keener consider is somewhat open to question. The writer has had some 50 years of experience with foundation rock and has seldom had the good fortune to secure absolutely undisturbed and homogeneous material.

Assumptions 8 and 9 deal with the modulus of elasticity, which is very difficult to estimate with accuracy for the following reasons: Many investigations have shown that concrete does not behave as a truly elastic material, the deformation under any given load being dependent more on "flow" or "creep" than elasticity. Messrs. Houk and Keener express the opinion that this characteristic can be allowed for adequately by the adoption of a somewhat lower "modulus of elasticity" than would otherwise be adopted for use in a technical analysis. Now, the modification that must be made is large and depends upon many considerations, to which the writer proposes to refer briefly.

It is only in comparatively recent years that the factors that govern the deformation of concrete under any given load have been studied, and much is still to be learned. Most information available in England is based on the experiments of R. D. Davies,[103] Oscar Faber,[104] and the work done at the Building Research Station, Watford, England, under W. H. Glanville.[105]

It has been shown: (1) That the deformation caused by the flow of concrete under sustained load may far exceed that due to the elastic modulus; (2) that this deformation depends upon the age of the concrete when loaded; (3) that it continues over a long period, reaching ultimately a "limiting" value; (4) that it is dependent upon the cement used; (5) that it is dependent upon the strength of the concrete mix; (6) that the degree of saturation with water has a marked effect in decreasing the "flow" of concrete that has been saturated with water; (7) that the grading of the aggregate also has a considerable influence; and (8) that computations should take account of changes in the volume of concrete caused by varying degrees of saturation, which also affect the coefficient of expansion and contraction due to changes of temperature.

The "flow" of concrete made with rapid hardening, or alumina, cements (see item (4)) is less than half that which occurs when normal portland cement is used, unless the concrete is not immersed in water, in which case the "flow" under any given load is greater for concrete made with rapid hardening and alumina cements.

The data in Table 18 are given by Glanville as an illustration of the effect on the modulus of elasticity due to varying the mix (item (5)), the concrete being made with normal portland cement loaded after twenty-eight days and kept in an atmosphere of uniform humidity.[106] The term "limiting" value (item (3)) denotes the modulus attained when "flow" ceases.

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[103] "Flow of Concrete Under Sustained Compressive Stresses," by R. D. Davies, *Proceedings*, Am. Concrete Inst., 1928, pp. 303–335.

[104] "Plastic Yield, Shrinkage and Other Problems of Concrete and Their Effect on Design," by Oscar Faber, *Proceedings*, Inst. of Civ. Engrs., 1927–1928, Pt. I, pp. 27–130.

[105] "The Creep or Flow of Concrete Under Load," by W. H. Glanville, *Building Research Technical Paper No. 12*.

[106] *Structural Engineer*, February, 1933, p. 57.

When it is complete, a dam will be subject to variation in the degree of saturation of the concrete (item (8)); that is, the portion above water level may become comparatively dry due to exposure on both faces to a hot sun, whereas the submerged portion will be partly saturated.

TABLE 18.—EFFECT OF MIX PROPORTIONING ON THE MODULUS OF ELASTICITY
(All Units, Pounds Per Square Inch)

Mix by weight	Modulus of elasticity	Effective Modulus Due to Influence of Flow	
		At twelve months	Limiting value
1 : 1 : 2	5.3×10^6	1.7×10^6	1.4×10^6
1 : 2 : 4	3.4×10^6	0.76×10^6	0.63×10^6
1 : 3 : 6	2.9×10^6	0.47×10^6	0.40×10^6

Moreover, the age of the concrete where the stress due to water level is imposed can scarcely be the same throughout the entire structure. The "effective modulus" for the lower part of a high dam probably will have nearly reached the "limiting value," whereas the age of the highest part may be only a year by the time the reservoir is filled, and the "effective modulus" applicable in the two cases may differ by a considerable amount.

Under "Stress Conditions," Messrs. Houk and Keener refer to Poisson's ratio effect, but experiments at the Building Research Station at Watford, on columns under compression, showed that they shrank laterally, when loaded, to about the same extent as they had when unloaded. As far as the writer is aware, little is known of the effect of varying degrees of saturation of concrete on changes of volume and their influence on the temperature coefficient, but Messrs. Houk and Keener, with their wide experience, will probably be able to cite some literature on the subject.

In making the foregoing remarks, it has not been the writer's intention to detract from the value of mathematical analysis, but to indicate that the foundations on which the mathematical structure rests are not so fully known as to justify too great a degree of refinement.

Under assumption 12 (heading "Load Conditions"), Messrs. Houk and Keener express the opinion that the silt contents of flood waters usually may be neglected in designing storage dams but may require consideration for relatively low ones. This opinion is not shared by all engineers. At the meetings of the Grand Barrage Section of the World Power Conference held at Tokyo, Japan, in 1929, no less an authority than the late Allen Hazen, M. Am. Soc. C. E., contributed a paper recommending that dams should be designed on the assumption that water weighed 100 lb per cu ft to allow for the possibility of silt pressure.

When constructing the Silent Valley Reservoir for the water supply of the Belfast and District Water Commissioners in Eire, the problem of sinking a trench to a great depth through "morainal" material saturated with water had to be faced; and, as much of it consisted of quartz ground to a powder, it was

apprehended that the pressure on the sides of the trench would be very high, calling for the adoption of special methods to give the necessary support to the sides of the trench.

It was ascertained by experience that this finely ground saturated material behaved as a liquid weighing 100 lb per cu ft, and the use of timber had to be abandoned and cast iron substituted.[107] In the case of low dams impounding heavily silt-laden water, the writer would adopt a liquid pressure of 100 lb per cu ft for the design, but agrees that it is only in very exceptional cases that this assumption is necessary for dams of any considerable height.

Messrs. Houk and Keener indorse the conclusion that it is seldom necessary to apply a straight-line pressure distribution from reservoir head to tailwater level to more than two thirds the area of the bank when considering uplift pressure. It is now usual to provide for drainage of the base of the dam close to the face, leakage water being discharged into galleries that may be only a little above tailwater level. It would appear that, if the drainage arrangements are adequate, the uplift pressure when foundations rest on non-porous rock would be small.

If the foundations of the dam penetrate rock to any considerable depth, the concrete becomes well bonded to the rock, owing to the irregularities in the sides of the trench, and the effective weight tending to resist "uplift" pressure is not only that of the dam itself but also that of the adjacent rock. The writer has sloped foundations toward the water face to increase security against sliding and so that the foundations at the water face may penetrate the rock to a sufficient depth to provide against possible "uplift" pressure, the slopes adopted varying from 1 in 12, for good sound rock, to 1 in 5 in the case of greasy shale.

Preparation of Foundations.—The paper by Messrs. Paul and Jacobs covers a wider field than might be gathered from the title and is full of instructive matter. Borings of small diameter which can be driven rapidly are of great benefit in enabling the engineer to select the best site for the dam; but they are likely to be misleading, as stated by the authors.

Experience at Silent Valley Reservoir[107] illustrates how one may be deceived. Preliminary borings were sunk across the valley in the usual manner, and the drills appeared to enter rock at depths that did not exceed 65 ft. Hence, a contract was let on the assumption that there would be no great difficulty with regard to the foundations. The rock was overlaid by morainal material which consisted of very fine sand and silt, saturated with water, in which were embedded large boulders left by the retreat of a glacier or ice sheet. It was these boulders that had been encountered, and not solid rock. The latter actually lay at a maximum depth of 196 ft from the surface, or 180 ft below the level of saturation of the morainal material. So numerous were the boulders that the contractor found it impracticable to drive piles to any depth.

The site would have been abandoned if it had not meant the sacrifice of a large sum of money which had already been spent on ancillary works before the true state of affairs was ascertained; and it was decided, therefore, to proceed with the work if it were possible to do so. Ultimately, the dam was successfully

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[107] "The Construction of the Silent Valley Reservoir, Belfast Water Supply," by G. McIldowie, *Proceedings*, Inst. of Civ. Engrs., Vol. 239, p. 498.

completed by the use of compressed air and novel methods of construction, which have been described elsewhere.[107]

The writer has not used large diameter borings for foundation exploration, but fully indorses the views of Messrs. Paul and Jacobs with regard to their advantages. The best course of all is undoubtedly to remove the overburden, or to excavate an exploratory trench if it is possible to delay letting the main contract until such work is completed. The writer agrees that abrupt changes in height above foundations are to be avoided, as cracks so frequently form at such points, and he has abandoned steps in favor of a sloping foundation.

Geological Problems of Dams.—The writer has read the paper by Mr. Crosby with great interest and considers it a masterly exposition of the subject, which should be in the hands of all engineers concerned with dam construction.

In England, geological conditions are generally unsuitable for the construction of masonry dams. Clays, soluble limestone, and chalk cover a large area, and most reservoirs are formed by means of embankments with a puddle core, material suitable for puddling generally being obtainable in proximity to the site.

Concrete Control.—Under the heading "Concrete Control: Cleanup, Curing, and Finishing," Mr. Tyler states that surface vibration of the concrete should be avoided because of the danger of bringing mortar to the surface and remixing it with accumulated water. The writer believes in using as little water as practicable when mixing concrete, and has found that more water is required for the successful operation of internal vibration than of surface vibration. He has obtained excellent results by spreading the "green" concrete in layers not more than 6 in. thick, the mix being so dry that moisture appears on the surface only after 2 or 3 min of vibration. The tool that is used is a pneumatic vibrator which has a mushroom-shaped head. It is not possible to use surface vibration in many instances, but a concrete dam affords an excellent opportunity.

In the sentence preceding "Conclusion," Mr. Tyler refers to the deflection of a 265-ft gravity dam $\frac{3}{8}$ in. upstream, against a rise in water level of 150 ft, due to seasonal temperature effects. The writer has noticed similar deflections upstream with increased water load in a case where temperature could scarcely account for it—namely, the Gorge Dam completed in 1936 for the supply of Hong Kong and Kowloon, China.

The dam is a composite structure with a maximum height of 275 ft above the stream bed, the water face being of concrete varying in thickness from 5 ft to 125 ft at stream-bed level, and the concrete portion is backed with hand-packed rock fill varying in thickness from 45 ft at 3 ft below top water level to 450 ft at stream-bed level.

Instruments were provided to measure the deflection from the vertical of the concrete portion of the dam under different conditions of water load, one being placed 93 ft above stream-bed level, the other recording the deflection at the top of the dam with reference to the level at which the lower instrument was placed.

Observations, which have now extended over a period of $3\frac{1}{2}$ years, have revealed the following facts, which are difficult to explain. The lower portion of the dam deflects toward the water as the reservoir level rises, whereas the

upper portion deflects in the opposite direction, the extent of these movements being as follows: In 1937 the movement from the vertical toward the water at 93 ft above stream bed was 0.16 in. with an increase in water level of 95 ft, and 0.14 in. with an increase in water level of 75 ft in 1939. There was a partial failure of the monsoon in 1938, the water level fluctuating irregularly, and the movements were small. The deflection from the vertical in the opposite direction at the top of the dam was more marked, amounting to 0.78 in. in 1937 and 0.7 in. in 1939 for the aforementioned increases in water level. Now if these movements were due to temperature changes, it is difficult to understand why the deflection of the upper and lower portions of the dam should be in opposite directions.

Construction Joints.—Mr. Steele is to be congratulated on the mass of valuable information contained in his paper. It was formerly the practice in England to make special provision for air slaking of any free lime in the portland cement before use.

When constructing the Vyrnwy Dam, the cement was placed in a lofty shed and caused to fall through the air from story to story, being aerated until the rise of temperature in setting did not exceed 3° F in 60 min or 2° in 15 min.[108] Another method was adopted by the writer's firm for the construction of the Alwen Dam, where the cement was placed in bins, provision being made to blow air upward through the cement. Perforated pipes were laid on the floor of the bins for that purpose. It was specified that aeration should continue for three weeks and for a period of not less than eight hours each day.

No provision was made for "transverse" joints in either dam, and the Vyrnwy Dam was free from all but hair cracks; but two vertical cracks appeared in the Alwen Dam at the abrupt change of section due to steps in the foundations. One of these cracks was difficult to detect and no percolation is visible, but slight percolation takes place through the other.

The practice of slaking the portland cement is not now followed in England. Instead, it is finely ground, and transverse joints have become desirable.

Under "Joint Spacing," Mr. Steele refers to the longitudinal joint in the Assuan Dam. In 1928 the Egyptian Government appointed an International Technical Commission, consisting of the late Hugh L. Cooper, M. Am. Soc. C. E. (for the United States), H. E. Gruner, M. Am. Soc. C. E. (for Switzerland), and the writer (for Great Britain), to consider the feasibility of increasing the storage by raising the dam a second time, and, in consequence of the Commission's Report, the dam was heightened by 9 m (29.5 ft).

The original dam was faced with cyclopean granite masonry, the "hearting" consisting of stones that could be handled, occupying 35 to 40% of the mass. The interstices were filled with 2 : 1 portland sand-cement mortar at the water face and near the foundations and 4 : 1 elsewhere, no transverse joints being provided.[109]

When it was decided to heighten the dam for the first time, it also became necessary to increase its width, the design of this work being entrusted to the late Sir Benjamin Baker, Hon. M. Am. Soc. C. E., who adopted the following

[108] "The Vyrnwy Works for the Water Supply of Liverpool," by G. F. Deacon, *Proceedings*, Inst. of Civ. Engrs., Vol. CXXVI, pp. 44 and 45.

[109] "Nile Reservoir, Aswan," by Sir Maurice Fitzmaurice, *Proceedings*, Inst. of Civ. Engrs., Vol. CLII.

procedure: A space of 30 in. was left between the old and the new work and, as this space was inclined to the vertical, steel rods $1\frac{1}{4}$ in. in diameter, spaced 1 m apart, were embedded in the old and penetrated the new work, the shuttering being removed when the concrete had set. After the new work had been completed, a period of two years was allowed to elapse so as to permit the equalization of temperature before the joint was filled with fine granite chips and grouted. No transverse joints were provided in the raised portion.[110]

The first problem with which the Committee was confronted was to determine whether the old and new work had been so well knitted together that the entire structure could be considered as homogeneous. The numerous sluice culverts which traversed the dam afforded the opportunity of internal inspection, and the junction between the grouted joint and the old and new work was cut back in the roof, sides, and floor of every culvert so as to permit an inspection by magnifying glass. The decision reached was that the dam could be considered as homogeneous, justifying calculations on that assumption. It was considered, however, that it would be too sanguine to assume that the entire mass would be homogeneous when the dam was heightened for the second time, and hence the additional support is given by massive buttresses that can expand and contract without stressing the main structure. Non-corrodible steel plates separate the buttresses from the body of the dam.

Mr. Steele refers to the transverse joints that were provided when the dam was raised for the second time, and these are spaced 23 ft apart, this distance being dictated by the position of the buttresses. He does not appear to favor metal stops to prevent percolation across the horizontal joints that occur between concrete which has already set and that which is superimposed; and, no doubt, if the greatest care is taken in preparing the surface of the old work in the manner that he describes, seepage should be avoided. However, the writer has so frequently observed seepage along these planes that metal stops were used in the case of the Gorge Dam already referred to.

An inclined and articulated reinforced-concrete diaphragm is provided at the water face, supported by buttresses spaced 12 ft 6 in. apart, projecting from the main concrete structure, which is referred to as the "thrust block." The diaphragm alone is relied on for watertightness and varies in thickness from 3 ft at the top to a maximum of 6 ft at a depth of 172 ft below overflow level. It is made of concrete containing 600 lb of cement per cu yd, in place. It is divided into panels by vertical transverse joints, spaced 25 ft apart, formed of bent copper plates 0.1 in. thick, immersed in a bitumen compound, the joints coinciding with the center of alternate buttresses. The concrete was placed in lifts of 20 ft, and longitudinal copper stops 0.1 in. thick, which varied in width from 2 ft to 1 ft, according to the water pressure, were built in at the top of each lift, projecting either 1 ft or 6 in. into the lift above.

It was found that the maximum rise of temperature above that of the atmosphere of the concrete was almost proportional to the quantity of cement used per unit volume, and amounted to 53° F as an average for concrete containing 600 lb of cement per cubic yard, in place, the average crushing strength being 350 tons per sq ft.

[110] "Protection of Downstream Rock Surface," by Sir Murdoch Macdonald, *Proceedings*, Inst. of Civ. Engrs., Vol. CXCIV.

Considerable economy was possible by reducing the quantity of cement used for the thrust block to 300 lb per cu yd, the crushing strength being 190 tons per sq ft and the maximum temperature rise being only 24° F above the atmosphere when setting.

The diaphragm is separated from the buttresses by means of special bituminous sheeting 0.2 in. thick, so that slight differential movement can take place without imposing stress. Similar sheeting was used to form the joint between adjacent panels and is interposed below the panels to separate them from the face concrete.

The space between the buttresses and behind the diaphragm communicates with galleries in the thrust block so that inspections can be made to ascertain whether leakage is taking place. It is gratifying to be able to record that only an occasional drip can be heard, which in all probability is due to condensation of the moist atmosphere.

In the case of the Gorge Dam the stresses due to water pressure are taken partly by the thrust block and partly by the rock fill. This composite structure was adopted partly because of the cheapness of Chinese labor and the immediate proximity of the granite quarries, which made possible the placing of hand-packed rock fill at a remarkably low figure. However, concrete crushing at 190 tons per sq ft affords an ample margin of safety to meet the stresses set up in a straight gravity dam of conservative design.

The writer would recommend the adoption of an articulated watertight diaphragm of a high-grade concrete, transmitting the thrust to the main dam, as considerable economy would be secured by adopting a concrete mix of lower grade for the latter, and all difficulties with regard to contraction or temperature cracks which permit percolation would be avoided.

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DISCUSSIONS

RECOMMENDED PRACTICE AND STANDARD SPECIFICATIONS FOR CONCRETE AND REINFORCED CONCRETE

Discussion

BY MESSRS. JOHN C. SPRAGUE, AND WALTER R. HNOT

JOHN C. SPRAGUE,[32] ASSOC. M. AM. SOC. C. E. (by letter).[32a]—In connection with that part of this important document which pertains to concrete proportioning, the following comments and data may be of some interest:

Drawing on information from various sources, for total water requirements for a 1-in. maximum-size aggregate, the data in Table 9 were compiled. Item 1 is from data published by Stanton Walker, M. Am. Soc. C. E., in 1939.[33] In item 2 the 33.5-gal item is for a well-rounded gravel and the 36.5-gal item is for a flat gravel.[34] A typical commercial gravel would be between these two extremes. The values for the Bureau of Reclamation[35] (item 3, Table 9) and unpublished tests by the writer (item 4) are significant in comparison. The

TABLE 9.—COMPARISON OF TOTAL MIXING WATER IN GALLONS REQUIRED PER CUBIC YARD OF NATURAL SAND AND GRAVEL CONCRETE

Item	Authority	Slump, in Inches					
		1–2	2–3	3–4	5–6	6–7	7–8
1	Stanton Walker	33	....	36	....	40	....
2	W. M. Dunagan	....	33.5 to 36.5	....	....	....	....
3	Bureau of Reclamation	....	....	35.5	....	....	....
4	The writer	34	35	36	38	....	40
5	W. A. Slater and Inge Lyse	....	....	....	40.5	....	....

NOTE.—This Report was published in June, 1940, *Proceedings*, Part 2. Discussion on this Report has appeared in *Proceedings*, as follows: September, 1940, by L. J. Mensch, M. Am. Soc. C. E.

[32] Development Engr., Dravo Corp., Keystone Sand Div., Pittsburgh, Pa.

[32a] Received by the Secretary October 3, 1940.

[33] "How to Design Concrete Mixtures," by Stanton Walker, *Proceedings*, Am. Concrete Inst., Vol. 35, 1939, p. 211.

[34] "The Application of Some of the Newer Concepts to the Design of Concrete Mixes," by W. M. Dunagan, *Journal*, Am. Concrete Inst., June, 1940.

[35] "Concrete Manual," U. S. Bureau of Reclamation.

value shown in item 5 is for a $\frac{3}{4}$-in. maximum-size[36] gravel, which is sufficiently close to the 1-in. size for practical purposes.

That the consistency of concrete mixes remains constant, regardless of richness, if the gradation of the aggregates and the water content per unit volume of concrete remains constant, is a reasonable assumption; this rule is known generally as the Slater-Lyse Rule. A comparison between the water

TABLE 10.—COMPARISON OF WATER REQUIREMENTS AND OTHER FACTORS FOR DIFFERENT TYPES OF FINE AGGREGATES

Material	Fineness modulus[a]	Percentage passing sieve 100	Water-cement ratio[b]	Cement factor[c]	Total mixing water[d]	Water-cement ratio[e]	Compressive Strength,[f] in Lb per Sq In.		
							7-day	28-day	90-day
Natural sand and gravel	2.97	2.0	0.87	1.1	30	0.77	....	....	....
Blended sand[g]	3.25	3.0	0.87	1.3	34	0.87	2,811	3,976	....
Natural sand	3.25	1.3	0.83	1.3	35	0.90	....	....	....
	2.97	2.0	0.87	1.3	35	0.90	2,862	4,023	4,408
Crushed sandstone sand	3.24	10.0	1.07	1.6	42	1.09	....	....	....
Crushed limestone sand	2.62	7.0	1.09	1.5	41	1.07	....	....	....
	3.24	10.0	1.09	1.6	42.5	1.09	2,442	3,206	3,681

[a] Fine aggregate. [b] Water-cement ratio for a mortar flow of 100%. [c] Cement factor for concrete with a water-cement ratio of 0.90 and a 2.5-in. slump. [d] In gallons per cubic yard of concrete, for a slump of 2.5 in. and a remolding effort of 20 drops. [e] Water-cement ratio required in concrete for a slump of 2.5 in. and a cement factor of 1.3. [f] Compressive strength of concrete for a cement factor of 1.33, a 3.5-in. slump, and the given water-cement ratio. [g] Blend of 80% natural sand and 20% crushed limestone sand.

requirements developed by the various sources shows very close agreement in all cases. Of course, some differences in grading are certain, although it is not believed that these differences will cause any appreciable fluctuation in water requirement as long as the ingredients are properly proportioned (see Table 10). One would naturally use a lower percentage of fine than of coarse sand in a balanced concrete mixture, with the net result that the fineness modulus of the combined aggregate would be substantially the same in all cases for a given maximum size of aggregate. In Table 10, all mortar is made of sand passing a No. 4 sieve, and the maximum size of all coarse aggregate in the concrete is 2 in., the fineness modulus being 7.86. Limestone is used as coarse aggregate except as indicated in the first line.

TABLE 11.—WATER REQUIRED FOR VARIOUS INGREDIENTS

Slump, in inches	Percentages by Weight			Gal per cu yd
	Cement	Sand	1-in. gravel	
1 to 2	23.5	6.0	3.0	30.9
3 to 4	23.5	6.5	3.5	32.6
5 to 6	23.5	7.0	3 5	33.1
7 to 8	23.5	9.0	3.5	35.3

Assume a hypothetical mix, by dry loose volume, of 1 : 2.25 : 4, involving quantities of 94 lb of cement, 200 lb of dry sand, and 396 lb of dry, 1-in. gravel. Next, assume the percentages of water by weight required by each of the ingredients of a concrete mixture to produce the slumps indicated in Table 11. On the strength of these assumptions and by using arbitrary specific gravities,

[36] "Compressive Strength of Concrete in Flexure as Determined from Tests of Reinforced Beams," by the late W. A. Slater and Inge Lyse, Members, Am. Soc. C. E., *Proceedings*, Am. Concrete Inst., Vol. 26, 1930, p. 831.

for cement and aggregate, of 3.1 and 2.65, it can be determined via the absolute-volume route that the cement requirement is approximately 5.6 sacks per cu yd of concrete for the 1-in. to 2-in. slump, ranging to approximately 5.45 sacks per cu yd for the 7-in. to 8-in. slump, and that the water-cement ratio ranges from about 5½ gal per sack of cement for the 1-in. to 2-in. slump to about 6½ gal per sack for the 7-in. to 8-in. slump. The net result of such a computation would be to produce the quantities of total mixing water shown in the last column of Table 11.

If these water quantities are compared, for the different slumps, with those in Table 9, it is noted that the former quantities are considerably lower. Therefore, it can be assumed that these quantities are too low in the light of the results shown in Table 9, which are in close accord and are the result of several independent investigations.

As the result of a number of tests with both natural and crushed aggregates, the writer[37] has found that there is a saving of 15% to 20% in cement content for 2-in. aggregate concrete as compared with 1-in. aggregate concrete, and that the total water requirement for the mixture is at least 15% less in the case of the larger aggregate. Table 10 shows the total water requirement, and other data, for 2-in. aggregate concrete for different types of fine aggregate. It will be noted that there is little difference in water and cement requirements for different finenesses of a given sand, although there are significant differences between different types of sand.

It is the opinion of the writer that Alternate A, paragraph 304, Chapter III, is definitely a step in the right direction. It is believed that the time will come when the contractor will be told what quality concrete is required, and it will be left to him to determine just how to achieve the desired ends, the responsibility for performance resting with him. If, as is required in many current specifications, the water-cement ratio, cement factor, and minimum strength are specified simultaneously, there is nearly always room for conflicting interpretations and misunderstandings. Where does the eventual quality of the concrete come in? Certainly, it is not necessarily a function of the strength of the concrete. Rather, it is a function of the initial quality of the ingredients; the water-cement ratio which controls the strength, durability, soundness, and other desirable qualities in concrete, assuming initially suitable ingredients; and other factors peculiar to the type and location of the structure.

WALTER R. HNOT,[38] JUN. AM. SOC. C. E. (by letter).[38a]—The building code of the American Concrete Institute, dated February 25, 1936, reads as follows:[39] "All bars in footing slabs, except the longitudinal reinforcement between loads in continuous slab footings, shall be anchored by means of standard hooks."

The new specifications of the Joint Committee do not mention the need of hooks. Does this mean that hooks in footings are no longer required unless needed by bond conditions? The writer would be interested to know exactly what the Committee had in mind regarding the general question of hooks.

[37] *Proceedings*, Am. Concrete Inst., Vol. 35, 1939, Table 4, p. 587.
[38] Structural Designer, Standard Oil Development Co., Elizabeth, N. J.
[38a] Received by the Secretary October 14, 1940.
[39] Building Code, ACI 501-36T, February 25, 1936.

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DISCUSSIONS

THEORY OF ELASTIC STABILITY APPLIED TO STRUCTURAL DESIGN

Discussion

BY LOUIS BALOG, ESQ.

LOUIS BALOG,[5] ESQ. (by letter).[5a]—For the solution of the general stability equation (Eq. 1), the knowledge of the stability coefficient k and the variation of the modulus of elasticity expressed by the modulus factor ζ are necessary. Stability coefficient k is a function of the dimensions, boundary, and loading conditions of the plate. The values of this coefficient are independent of the material constants E and m. The determination of the k-values for the various conditions of support and loading is a mathematical procedure that can be solved without a knowledge of the characteristics of the plate material.

For simply supported rectangular plates subject to direct stresses along the depth of the plate, Eq. 3 gives the approximate minimum values of coefficient k. In Table 1 the k-values for uniform shear along the four edges of the plate are given. Table 2 gives the reduction of the k-values of pure bending by the simultaneous action of shear.

A frequently occurring design case is that involving bending, axial stress, and shear. For the most economical design, the distribution of the material in sections composed of shapes should be such that bending and axial stresses produce stresses equal and of the opposite sign in the extreme fibers of the sections. This assures that the flange areas are minimum and the allowable web slenderness is maximum. The k-values for pure bending, which are the largest and which will be reduced the least by shear, apply for the web design.

It is not always practicable, however, to satisfy the criteria of the most economical design. Constant sections and rolled beams also often occur as members subject to bending, axial stress, and shear. In these cases k will be smaller than for pure bending, and the reduction of k due to shear will be larger than that given in Table 2. The authors' diagrams for the allowable slenderness ratios cannot be used.

NOTE.—This paper by Leon S. Moisseiff and Frederick Lienhard, Members, Am. Soc. C. E., was published in January, 1940, *Proceedings*.

[5] Engr. with Leon S. Moisseiff, Cons. Engr., New York, N. Y.

[5a] Received by the Secretary October 8, 1940.

The slenderness-ratio diagrams are based on the minimum value of k for pure bending in simply supported plates, with the corresponding shear reduction. If the width-to-depth ratio of the plate is smaller than the β-ratio to which the minimum k-value corresponds, k increases. In the stress-distribution range between pure bending and uniform compression, k decreases and the reduction of k due to the simultaneous action of shear increases. Safety, uniformity of the safety factors of the parts of the structure, and economical considerations require that the allowable plate slenderness shall be determined with due consideration of the width-to-depth ratio β and the actual stress distribution wherever these are significantly different from those on which the allowable slenderness ratio diagrams were based.

In structures, the support condition of the plates is seldom clearly defined. Deviations from the true plane plate surface may more than offset the advantages of restraint. It is prudent, therefore, to use, for most structural plates, the buckling coefficients of simply supported plates.

The computation of the rigorous k-values is a laborious operation. Simple relations between the ratio β and the loading conditions, giving sufficiently close approximations for the buckling coefficient k, are of interest only in practical design. Such equations for the stress-variation range from uniform compression to pure bending may be given as follows:

Let σ_1 be the compressive stress at the toe of the top flange angle and σ_2 the compressive or tensile stress at the toe of the bottom flange angle, and the ratio $\frac{\sigma_2}{\sigma_1}$ will express linear stress variation in the depth of the plate. Then the basic loading conditions of the plate, uniform compression, triangular stress distribution, and pure bending are expressed by: $\frac{\sigma_2}{\sigma_1} = 1$; $\frac{\sigma_2}{\sigma_1} = 0$; and $\frac{\sigma_2}{\sigma_1} = -1$, respectively. It is noted that $\frac{\sigma_2}{\sigma_1} = 1 - \alpha$, with α defined by Eq. 2.

The k-values for the case of $\frac{\sigma_2}{\sigma_1} = 1$ or $\alpha = 0$ (uniform compression) can be obtained from the simple equation, when $\beta \leqq \sqrt{2}$,

$$k = \left(\beta + \frac{1}{\beta}\right)^2 = k_u \qquad (42)$$

For the case in which $\frac{\sigma_2}{\sigma_1} = 0$ or $\alpha = 1$ (triangular stress distribution), when $\beta \leqq 1$,

$$k = 1.91\, k_u = k_t \qquad (43)$$

When $\beta > 1$, the coefficient of $k_t = 7.64$. For the stress variation range between $\frac{\sigma_2}{\sigma_1} = 1$ (uniform compression) and $\frac{\sigma_2}{\sigma_1} = 0$ (triangular stress distribution), Professor Chwalla proposes:[6]

When $\beta \leqq 1$

$$k = k_u \frac{2.1}{1.1 + \frac{\sigma_2}{\sigma_1}} \qquad (44a)$$

[6] "Erläuterungen zum Normblattentwurf DIN E 4114," Stahlbau-Verband, Berlin, 1939, p. 9.

and, for $\beta > 1$,

$$k = \frac{8.4}{1.1 + \frac{\sigma_2}{\sigma_1}} \qquad \text{(44b)}$$

For $\frac{\sigma_2}{\sigma_1} = -1$ or $\alpha = 2$ (pure bending), when $\beta \leqq 0.67$,

$$k = 15.870 + \frac{1.87}{\beta^2} + 8.6\,\beta^2 = k_b \qquad \text{(45)}$$

When $\beta > 0.67$, the coefficient $k_b = 23.9$. For the stress variation range between $\frac{\sigma_2}{\sigma_1} = 0$ (triangular stress distribution) and $\frac{\sigma_2}{\sigma_1} = -1$ (pure bending), the German Railroads specify[7]

$$k = \left(1 + \frac{\sigma_2}{\sigma_1}\right) k_t - \frac{\sigma_2}{\sigma_1} k_b + 10 \frac{\sigma_2}{\sigma_1}\left(1 + \frac{\sigma_2}{\sigma_1}\right) \qquad \text{(46a)}$$

or, if the stress variation is expressed by α,

$$k = (2 - \alpha)\, k_t - (1 - \alpha)\, k_b + 20 - 30\,\alpha + 10\,\alpha^2 \qquad \text{(46b)}$$

The k-values obtained by means of the foregoing simple equations will indicate whether or not the slenderness ratio diagrams of the authors can be used in design cases different from pure bending. The coefficients apply when the direct stresses are not accompanied by shear. Shearing stresses reduce the stability of the plate submitted to direct stresses. This reduction is the smallest for pure bending and the largest for uniform compression.

It is of practical importance, for design purposes, to establish a sufficiently accurate and simple functional relation between the interdependent critical stress combinations σ_c and τ_c. Such relations can be derived from the following considerations: If one of the stresses in the combination is large the other must be small. If the plate is subject only to direct stresses, $\sigma_{cr} = \sigma_c$; and, if only shearing stresses act, $\tau_{cr} = \tau_c$. The ratios $\frac{\sigma_c}{\sigma_{cr}}$ and $\frac{\tau_c}{\tau_{cr}}$ may have any value between zero and one. This can be expressed by a general equation of the form

$$\frac{\sigma_c}{\sigma_{cr}} = f\left(\frac{\tau_c}{\tau_{cr}}\right) \qquad \text{(47)}$$

in which the relation changes with β.

Plotting $\frac{\sigma_c}{\sigma_{cr}}$ against $\frac{\tau_c}{\tau_{cr}}$, curves are obtained intersecting both coordinate axes at unity. For pure bending and shear, the curve should be perpendicular at, and symmetrical about, both axes because the reversal of the sign of either moment or shear does not change the character of the stress condition of the plate. For uniform compression and shear, the change of the sign of the shear does not alter the stress condition; hence, the curve should be perpendicular at and symmetrical about the $\frac{\sigma_c}{\sigma_{cr}}$-axis. This curve, however, cannot be sym-

[7] "Berechnungsgrundlagen für stählerne Eisenbahnbrücken," Berlin, 1937, p. 60.

metrical about, nor can it have a rectangular intersection with, the $\frac{\tau_c}{\tau_{cr}}$-axis because reversal of the sign of the direct stresses changes the stress condition of the plate. These considerations suggest, for the approximation of the critical stress combinations, the following power relation:

$$\frac{\sigma_c}{\sigma_{cr}} = 1 - \left(\frac{\tau_c}{\tau_{cr}}\right)^n \quad \text{.......................(48)}$$

The combined axial compression and torsion tests made with a great number and variety of thin-walled cylinders by F. J. Bridget, C. C. Jerome, and A. B. Vosseller[8] give 2.63 for the average value of n. The results of these tests and the results of the theoretical investigations for combined uniform compression and shear, made by Southwell and Skan (7),[8a] Wagner (11), Schmieden (34), Wansleben (35), Chwalla (40), and Iguchi (51), are all covered sufficiently well for design purposes by the general empirical relation Eq. 48, if the value of n is properly selected. The value of exponent n is a function of β.

For simply supported rectangular plates under the action of uniform compression and shear, $n = 2$ gives the best approximation of the relation between the stress combinations σ_c and τ_c, for all values of $\beta \leqq 1$; thus:

$$\frac{\sigma_c}{\sigma_{cr}} = 1 - \left(\frac{\tau_c}{\tau_{cr}}\right)^2 \quad \text{.......................(49)}$$

and so the percentage of k available for direct stress is

$$k_d = 1 - \left(\frac{\tau}{\tau_{cr}}\right)^2 \quad \text{.......................(50)}$$

Interpolating between the values of k_d obtained from Eq. 50 and the values given in Table 2, safe values for k_d will be obtained for the direct-stress variation range between uniform compression and pure bending. These values are given in Table 12, which shows that the reduction percentages k_d increase considerably as compression extends over the larger part of the section.

It is interesting to note that the reduction of bending stresses is similar to the reduction of pure axial tension stresses by the simultaneous action of shear. Tests made by E. A. Davis[9] determine the tension and torsion (shear) stress combinations that caused the failure of cylindrical bars and the ultimate values of the stresses when they were applied separately. Plotting from these tests the ratios $\frac{\sigma_c}{\sigma_u}$ against $\frac{\tau_c}{\tau_u}$, a curve similar to the curve for k_d represented by Table 2 will be obtained. The approximate power relation, Eq. 48, for both cases takes the form

$$\frac{\sigma_c}{\sigma_{cr}} = \frac{\sigma_c}{\sigma_u} = \left[1 - \left(\frac{\tau_c}{\tau_{cr}}\right)^2\right]^{0.5} \quad \text{..................(51)}$$

[8] "Some New Experiments on Buckling of Thin-Wall Construction," by F. J. Bridget, C. C. Jerome, and A. B. Vosseller, *Transactions*, A. S. M. E., Vol. 56, 1934, pp. 569–578.

[8a] Numerals in parentheses, thus (7), refer to corresponding items in the Bibliography in the Appendix of the paper.

[9] "Combined Tension-Torsion Tests on a 0.35 Per Cent Carbon Steel," by E. A. Davis, *Transactions*, A. S. M. E., Vol. 62, October, 1940, p. 577.

Eq. 51 or the values in Table 2 can be used for k_d in the stress-variation range between pure bending and uniform tension with a combination of shear.

With the knowledge of the k-values and the plate material constants E and m, the critical buckling stresses can be determined as long as they are smaller

TABLE 12.—REDUCTION OF k (EQS. 42 TO 46) DUE TO SIMULTANEOUS ACTION OF SHEAR

$\frac{\sigma_2}{\sigma_1}$	VALUE OF k_d FOR THE RATIOS $\frac{\tau}{\tau_{cr}}$ EQUAL TO:									
	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
1.0	0.990	0.960	0.910	0.840	0.750	0.640	0.510	0.360	0.190	0.000
0.9	0.990	0.961	0.912	0.843	0.755	0.646	0.518	0.370	0.200	0.000
0.8	0.990	0.961	0.913	0.846	0.759	0.652	0.526	0.379	0.210	0.000
0.7	0.990	0.962	0.915	0.849	0.764	0.658	0.534	0.389	0.220	0.000
0.6	0.990	0.962	0.916	0.852	0.768	0.664	0.542	0.398	0.230	0.000
0.5	0.990	0.963	0.918	0.855	0.773	0.670	0.550	0.408	0.240	0.000
0.4	0.990	0.963	0.919	0.858	0.777	0.676	0.558	0.417	0.250	0.000
0.3	0.990	0.964	0.921	0.861	0.782	0.682	0.566	0.427	0.260	0.000
0.2	0.990	0.964	0.922	0.864	0.786	0.688	0.574	0.436	0.270	0.000
0.1	0.990	0.965	0.924	0.867	0.791	0.694	0.582	0.446	0.280	0.000
0.0	0.990	0.965	0.925	0.870	0.795	0.700	0.590	0.455	0.290	0.000
−0.1	0.990	0.966	0.927	0.873	0.800	0.706	0.598	0.465	0.300	0.000
−0.2	0.990	0.966	0.928	0.876	0.804	0.712	0.606	0.474	0.310	0.000
−0.3	0.990	0.967	0.930	0.879	0.809	0.718	0.614	0.484	0.320	0.000
−0.4	0.990	0.967	0.931	0.882	0.813	0.724	0.622	0.493	0.330	0.000
−0.5	0.990	0.968	0.933	0.885	0.818	0.730	0.630	0.503	0.340	0.000
−0.6	0.990	0.968	0.934	0.888	0.822	0.736	0.638	0.512	0.350	0.000
−0.7	0.990	0.969	0.936	0.891	0.827	0.742	0.646	0.522	0.360	0.000
−0.8	0.990	0.969	0.937	0.894	0.831	0.748	0.654	0.531	0.370	0.000
−0.9	0.990	0.970	0.939	0.897	0.836	0.754	0.662	0.541	0.380	0.000
−1.0	0.990	0.970	0.940	0.900	0.840	0.760	0.670	0.550	0.390	0.000

than the proportional limit of the plate material. For higher critical stresses, the variation of the modulus of elasticity as expressed by the ζ-values must be considered. The critical stresses in structural design, as a rule, are higher than the proportional limit, and the ζ-values assume similar importance to buckling coefficient k in the design of plates.

The k-values are obtained by analytical methods; the ζ-values can be determined only empirically. In Table 3 the authors give ζ-values for structural carbon and silicon steel, and for aluminum alloy. These ζ-values were derived from curves approximating the critical stresses of tested columns.

The multiplier $\epsilon = \frac{E_x}{E}$ of the Euler strength of columns establishes a functional relation $\epsilon = f(\sigma_{cr})$ between the critical stress and elasticity modulus. The integral curve of this function represents the compressive stress-strain relation, $\sigma_{cr} = f(\mu)$. This stress-strain relation is materially different from the compression stress-strain relation, $\sigma_c = f(\Delta_c)$, which expresses the deformation of a compression-test specimen with increasing stress. The stress-strain curves of steels and aluminum, $\sigma_c = f(\Delta_c)$, as determined by Erich Siebel and Anton Pomp[10] are higher than the tension stress-strain curves, $\sigma_t = f(\Delta_t)$, whereas the curves, $\sigma_{cr} = f(\mu)$, representing the relation of the buckling strains to the critical stress of column specimens of various lengths, are always below the tension stress-strain diagrams.

[10] "Die Ermittlung der Formänderungsfestigkeit von Metallen durch den Stauchversuch," by Erich Siebel and Anton Pomp, *Mitteilungen aus dem Kaiser-Wilhelm-Institut für Eisenforschung*, Vol. 9, Stahleisen, Düsseldorf, 1927, p. 164.

Column strength depends on the magnitude of bending that accompanies compression. As the column approaches failure, the compressive strains change to tensile strains in one part of the section. At failure, the latter strain attains greater extreme value than the strain at the extreme compression fiber. The resulting strain, characteristic for the strain condition at buckling, is represented by the relation $\sigma_{cr} = f(\mu)$. The effect of the tensile strains on this relation suggested to the writer a connection between the tension stress-strain and the relations $\sigma_{cr} = f(\mu)$ of various metals. If the relation between the tension stress-strain curve and the curve of a metal, $\sigma_{cr} = f(\mu)$, is known, the latter curve of another metal can be derived from its tension stress-strain diagram.

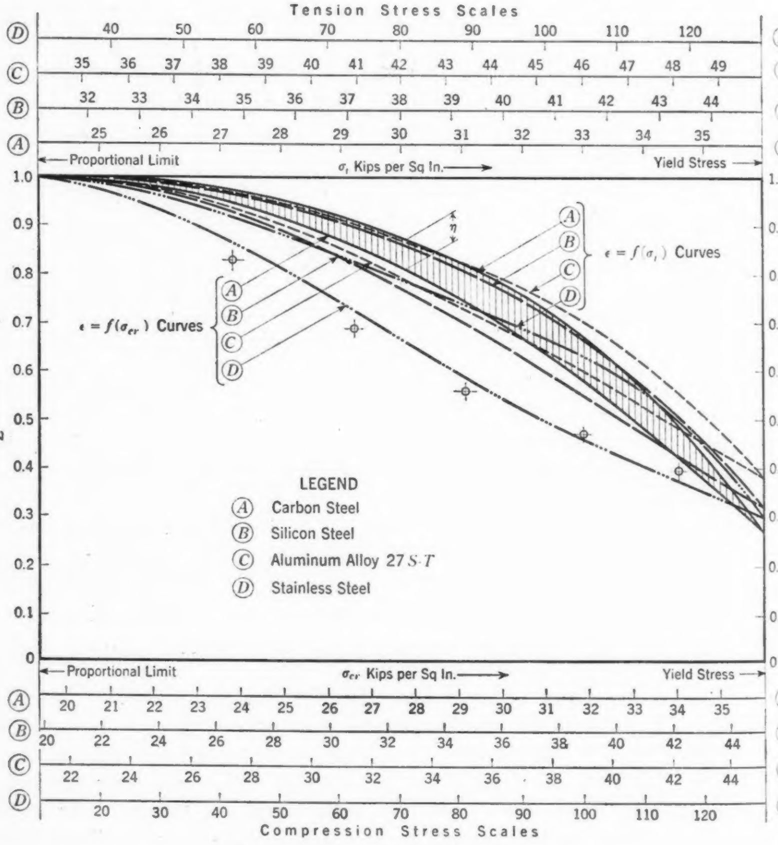

FIG. 10.—$\epsilon = f(\sigma_t)$ AND $\epsilon = f(\sigma_{cr})$ CURVES

The relative magnitude of the yield stresses and the relative width of the stress ranges between the proportional limit and the yield stress influence the shapes of the stress-strain curves greatly—hence the shapes of the curve of the elasticity modulus variation. To study the relation between the elasticity

modulus variation in tension and compression, a representation must be found that is suitable for the comparison of the variations.

As a rule, the width of the stress range between the proportional limit and yield stress is different for tension and compression and varies widely for various metals. If these different stress ranges, between the proportional limit and yield stress, are represented for various metals by the same length, a suitable scale is found for the comparison of the variation of the elasticity moduli. In this representation each metal has its own stress scales along the abscissa axis of the coordinate system (see Fig. 10). The ordinates represent the ratio between the varying and constant values of the elasticity moduli. The scale of the ordinates is the same for all metals.

In this representation the differential curves $\epsilon = f(\sigma_{cr})$ of the typical tension stress-strain diagrams of the three metals investigated by the authors are nearly identical except for the vicinity of the yield stress. The curves of these metals, $\epsilon = f(\sigma_{cr})$, plotted from Table 3 are flatter, more apart, and follow the general trend of the tension modulus variation curves.

Fig. 10 indicates that the shape of the curves is independent of the constant values of the elasticity moduli and that the tension and the modulus variation curves of each metal, $\epsilon = f(\sigma_{cr})$, form a loop. The depth of these loops widens with the increase of the yield stress and the breadth of the range between the proportional limit and the yield stress. The depth of the loops is about in the proportion as the product of the ratios of yield stress and width of the yield range of carbon steel and the width of the yield range and yield stress of the other metals.

From this relation the elasticity modulus variation $\epsilon = f(\sigma_{cr})$ can be derived for a given metal from its tension stress-strain diagram if the proportional limit of the stress-strain relation $\sigma_{cr} = f(\mu)$ is known. For metals with long yield range and low proportional limit, this can be taken as equal to the proportional limit in tension, or it can be lowered arbitrarily to obtain about 20% wider yield range for compression. The modulus variation $\epsilon = f(\sigma_{cr})$ will then be obtained by the reduction of the values of $\epsilon = f(\sigma_t)$. The reduction factor is

$$\psi = \frac{\sigma_{yc}\,X_{yr}}{\sigma_{yx}\,C_{yr}}\,\eta \qquad (52)$$

in which σ_{yc} = yield stress of carbon steel; C_{yr} = stress range between the proportional limit in compression and yield stress of carbon steel; σ_{yx} = yield stress of the given metal; X_{yr} = stress range between the proportional limit in compression and yield stress of the given metal; and η = the ordinates of the shaded area between curves *A*, Fig. 10.

The reduction factor for silicon steel is $\psi = \frac{36 \times 25.12}{45 \times 16.61}\,\eta = 1.21\,\eta$. Reducing the $\epsilon = f(\sigma_t)$-curve *B* (Fig. 10) by $1.21\,\eta$, the $\epsilon = f(\sigma_{cr})$-curve *B* is obtained. This curve is practically identical with the curve representing the ϵ-values given by the authors for silicon steel in Table 3.

In a similar manner, for the aluminum alloy, $\psi = \frac{36 \times 24.07}{45 \times 16.61}\,\eta = 1.16\,\eta$, and the reduction of the $\epsilon = f(\sigma_t)$-curve *C* (Fig. 10) results in the $\epsilon = f(\sigma_{cr})$-curve *C* representing similar ϵ-values given by the authors in Table 3.

Evaluation of the tests made by Howard W. Barlow, Henry S. Stillwell, and Ho-Shen Lu[11] with stainless steel columns gave the $\epsilon = f(\sigma_{cr})$-values indicated by the five plotted points in Fig. 10. The metal used in these tests had a proportional limit of 30 kips per sq in. in tension and 10 kips per sq in. in compression. The yield stress was 130 kips per sq in. The writer reduced the $\epsilon = f(\sigma_t)$-curve D (Fig. 10) by $\psi = \dfrac{36 \times 120}{130 \times 16.61}\,\eta = 2.0\,\eta$ and obtained the $\epsilon = f(\sigma_{cr})$-curve D. The general trend of this curve agrees with the test results, and the differences are about 5%.

In Eq. 52, reduction factor ψ is based on carbon steel. This factor, of course, can be based on any other steel for which the relations $\epsilon = f(\sigma_t)$ and $\epsilon = f(\sigma_{cr})$ are known. The writer does not claim that reduction factor ψ applies rigorously in every case. More extensive studies are required to refine the expression for the reduction factor.

The significance of obtaining $\epsilon = f(\sigma_{cr})$ from the tension test is that to undertake extensive column testing programs for each metal of different proportional limit and yield strength is not practical. Tests indicating the stress-strain relation in tension, on the other hand, are simple, inexpensive, and rapid. The tension test shows more clearly and consistently the plastic behavior of metals than any other. In testing for buckling there are endless factors that influence the test results. The compression member in the structure is mostly in an entirely different stress condition than the test specimen. Imperfections in the fabrication and erection, the moments at the ends, and along the member contribute more to the uncertainties of the factor of safety than deviations in the ϵ-values. The design for buckling can be based on the reduced tension-stress-strain diagram.

The authors propose $\zeta = 0.5\,(\epsilon + \sqrt{\epsilon})$ as the modulus factor for plates for all stress conditions. Although this is a practical generalization, it should be kept in mind that the modulus variation is different for homogeneous stress conditions, like uniform compression, and nonhomogeneous stress conditions, like bending. For stiffened plates, the modulus variation changes not only with the stress condition but also with the make-up of the plate. Poisson's ratio is 0.3 for the elastic part and 0.5 for the plastic part of the deformations. It appears that only tests can show the stiffness variation of plates of more complicated arrangement under the various stress conditions. Such research is available for types not used in bridge and building construction. The authors' proposal meets the requirements of structural design safely.

By deriving simple and practical rules for safe and economical design of the most complex structures, the authors have made an outstanding contribution to promote progress in structural design. The writer has attempted to suggest a way to remove obstacles to a general use of the material presented in the paper.

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[11] "Strength Investigations of Thin Stainless-Steel Sections," by Howard W. Barlow, Henry S. Stillwell, and Ho-Shen Lu; presented at the 1940 Annual Meeting of the Institute of the Aeronautical Sciences.

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DISCUSSIONS

PERMISSIBLE COMPOSITION AND CONCENTRATION OF IRRIGATION WATER

Discussion

By LEON D. BATCHELOR, ESQ.

LEON D. BATCHELOR,[10] ESQ. (by letter)[10a].—There has been much confused thinking on the subject of permissible composition of irrigation water, which could be materially clarified by a careful consideration of this article. The author is to be commended for his summarization of his own research and that of others on this important subject.

Even greater emphasis might properly have been placed by the author on the rôle of the native salts in the soil and subsoil, prior to irrigation, in causing subsequent injury to crops. The movement of such salts to the root zone of crops, by capillary rise of the soil moisture from a perched water table, is recognized by Professor Kelley, as it often has been by various students of this subject. Unfortunately, precise data relative to the native salt content of the soil do not exist in most irrigated sections. A notable exception to this is the Imperial Valley, in California, in which a survey was made prior to any irrigation, and salt determinations were made of the surface soil, lower root zone depths, and, in two cases, the deep subsoil to a depth of 22 ft. The results of this investigation were published[11] in 1902, long before it was generally recognized that the salt and drainage problem was to be a serious consideration in the Imperial Valley. Tests of rate of percolation on various soil types were also reported in this bulletin. These tests, together with the salt determinations which showed enormous amounts of native salt in the subsoil, implied, in advance, much of the difficulty which actually has been encountered in this area. These early studies also enable one to evaluate, to some extent at least, the part played by the irrigation water on one hand and by the native salts of the soil on the other hand in bringing about a salt problem in the area. Unfor-

NOTE.—This paper by W. P. Kelley, Esq., was published in April, 1940, *Proceedings*. Discussion on this paper has appeared in *Proceedings*, as follows: September, 1940, by Messrs. Carl S. Scofield, Walter W. Weir, and Robert S. Stockton.

[10] Director, Univ. of California Citrus Experiment Station, Riverside, Calif.

[10a] Received by the Secretary August 9, 1940.

[11] Discussion by E. W. Hilgard and G. W. Shaw of "Lands of the Colorado Delta in the Salton Basin," by F. J. Snow, *Bulletin No. 140*, Univ. of California, 1902.

tunately, such fundamental preliminary data concerning irrigated areas are not commonly available.

In the explanation of the effect of ordinary irrigation in leaching the salts below the root zone of plants, the sound procedure, in general, is emphasized that the more saline the water is the more water should be applied in order that leaching may occur every time water is applied. It is the belief of the writer that this could be overdone beyond the point of accomplishing the desired end of avoiding the accumulation of salt in the root zone. First, it seems doubtful to the writer whether irrigation water is ordinarily applied to most crops in such quantities that the amount of water is no more than enough merely to wet the soil of the root zone. The opinion is held that in the vast majority of instances, in the case of annual crops and shallow-rooted tree crops, the casual irrigation causes a notable amount of leaching. As pointed out by data prepared by the author, the rains are an important factor in leaching the salts out of irrigated lands. If leaching is promoted by the use of extra irrigation water in addition to normal rainfall, it should be done only when precise data show its need. Otherwise, an unnecessary loss of nitrates from the soil may be the result of an uncalled for attempt to reduce the salt concentrations.

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DISCUSSIONS

FLOOD-PROTECTION DATA

PROGRESS REPORT OF THE COMMITTEE

Discussion

BY GORDON R. WILLIAMS, ASSOC. M. AM. SOC. C. E.

GORDON R. WILLIAMS,[10] Assoc. M. Am. Soc. C. E. (by letter).[10a]—The Committee's condemnation of the method of computing average annual flood damage by use of stage-damage and flood-frequency curves should not go undisputed. The method admittedly has its limitations; but it is at least a logical approach to the problem of determining the economic justification of flood-control projects, and when modified by the dictates of good judgment and experience should give the best results obtainable with the data available.

The Committee's suggestions for an alternate procedure are indefinite. Apparently, it is suggested that the present damage which would result from known floods should be added and the total divided by the record of experience to obtain the annual damage. With a very long flood record such a procedure should give results not greatly different from those obtained by the use of a frequency curve. However, floods and corresponding damages tend to come in irregular cycles and, if the foregoing method is used with a short record of experience, a distorted result may be obtained. Methods of overcoming the limitations of short records through use of modified frequency curves are discussed herein. To examine an extreme case, assume that the damages that would result from floods of the last five years at damage centers on eastern rivers are added up and divided by five. The indicated average damage will be large; but will it be indicative of the average damage in the next twenty-five or fifty years, a period in which the flood protection must pay for itself? Economic justification is determined by comparing the average annual charge in the amortization period against the portion of the expected average annual damage prevented in that period. The latter can be determined only by the best possible analysis of the damages to be expected per flood and the expected chance of occurrence of the floods.

NOTE.—This Progress Report of the Committee on Flood-Protection Data was published in April, 1940, *Proceedings*. Discussion on this Report has appeared in *Proceedings*, as follows: June, 1940, by Waldo E. Smith, M. Am. Soc. C. E.; and September, 1940, by O. J. Todd, M. Am. Soc. C. E.

[10] Associate Hydr. Engr., U. S. Engr. Office, Baltimore, Md.

[10a] Received by the Secretary September 24, 1940.

Consider in outline some of the steps necessary in computing average annual damage, taking first the stage-damage curve. Such a curve has somewhat limited use in evaluating agricultural damage; but it may be modified to take care of special conditions and may be used with seasonal flood-frequency curves to compute expected annual damages. Stage-damage curves are distinctly useful in evaluating urban damage if derived in a painstaking and logical manner. The first step is to take a census of the number and classes of structures to be inundated at each stage. By detailed study of a representative number of buildings, the average damage likely to occur in each class of structures for various degrees of flooding may be determined. For example, basement damage, first-floor damage, and second-floor damage for dwellings of various classes can be determined. Industrial plants and utilities must be treated as individual problems. Allowance must be made for time of flood warning and the possibility that goods and machinery may be moved above the damage line. It should be kept in mind that the stage-damage curve is to be used for the purpose of evaluating future recurring damage and that past damages should be used only as a guide in evaluating possible future damage. If bridges, highways, and industries have been moved to higher elevations, they are no longer subject to future damage at the same elevation as before. Enhancement of property values, largely due to flood protection, should be treated separately and should not be confused with damages prevented.

To determine average annual damage from a stage-damage curve, which gives the damage per flood, a damage-frequency curve is constructed by plotting the damage per flood against its expected chance of occurrence in any year, or season in the case of agricultural damage. The area under the damage-frequency curve represents the average annual damage. In plotting the curve the percentage chance of occurrence is obtained from the frequency curve, derived for either stages or discharges. If channel conditions have not changed over the period of record, as far as the high-water control is concerned, it is advisable to use only the stage-frequency curve because: (1) The stage must be determined for use with the damage curve; (2) the use of stage in the frequency curve eliminates errors inherent in the determinations of high-water, stage-discharge curves; and (3) damage stages caused by ice jams are taken into consideration.

Many will argue that the method is invalidated because the frequency curve is not reliable. On small rivers with short records of stream flow, frequency curves are not dependable; but on large rivers where most important damage centers are located, frequency curves are reasonably accurate within the limits significant to a damage study. There are often continuous records of thirty to fifty years, which, when combined with historical data extending back one hundred years or more, give good results. If the record at the damage center is short, it may be modified by comparing it with generalized frequency curves constructed from long records at other places. In this connection it should be borne in mind that to justify flood control at a damage center there must be frequent if not excessive flood damage. In other words, it is not as necessary to determine the 500-yr flood damage accurately as it is to determine the 5-yr to 50-yr damage. The portion of the annual damage for justified projects

that may be due to floods rarer than the 100-yr or 1% chance flood is usually a small fraction of the total annual damage. The periodicity of frequent flood stages and damages, which are the critical ones in determining justification, can be determined with a reasonable degree of accuracy at most damage centers on important rivers.

A relatively few years ago statistical methods were thought to be the panacea for most deficiencies in hydrologic data. They have now been discredited as a method of determining the magnitudes of rare or largest probable floods; but such methods have their place in the economic side, if not in the purely hydrologic side, of flood problems.